A Shot Noise Model For Financial Assets

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In this article we propose and study a model for stock prices which allows for shot-noise effects. This means that abrupt changes caused by jumps may fade away as time goes by. This model is incomplete. We derive the minimal martingale measure in discrete and continuous time and discuss the associated hedging strategy. Finally, a simulation study is included to show that our model is able to produce smile effects.

Keywords: Shot-Noise Component; Jump Diffusion; Minimal Martingale Measure.

1 Introduction

In financial markets information often comes as a surprise. This usually leads to abrupt changes in stock prices, may it be upward or downward. Quite often such situations are used by investors to adjust their portfolios. For example, an upward jump may lead to profit-taking while a downward jump may encourage new investors to buy the asset. In each case this results in a so-called shot-noise effect, i.e., as time passes by, the jump completely or at least partially fades away.

A by now classical approach to incorporate jumps in a stock price model is via so-called jump-diffusion models. See, e.g., Merton (1973). Here the price follows a diffusion process interrupted by jumps. Since no fade-away-components are included, the effect of a jump persists forever. As we have argued above this may be unrealistic so that shot-noise models constitute useful extensions of jump diffusions. Viewed from another perspective, shot-noise models build an efficient class of models to study market imperfections, such as overreaction or irregular behaviour due to, for example, illiquidity.

As to credit-worthiness of a company shot-noise effects may arise due to incomplete information effects as pointed out by Collins-Dufresne et al. (2003). A prominent example is Enron's collapse followed by its bankruptcy.

Also, Klüppelberg and Kühn (2004) consider shot-noise models and show that as a limit of shot-noise processes a fractional Brownian motion is obtained. For a short introduction to shot-noise processes we refer to Bondesson (1988). Further applications of shot-noise processes may be found in, for example, Dassios and Jang (2003) and Gaspar and Schmidt (2006).

The paper is organized as follows. In section 2 we propose and discuss in detail the new model. As we shall see the model consists of three major parts:

- a continuous diffusion

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- a marked point process
- a shot-noise component

The resulting model describes an incomplete market. For pricing of derivatives we need to determine a martingale measure. In this paper we study the associated minimal martingale measure. In section 3 we first compute the minimal martingale density for the case when trading takes place in discrete time. By considering finer and finer grids we come up, in the limit, with the density in continuous time. The minimal martingale measure in this framework is generally a signed measure and we provide several conditions which guarantee that the minimal martingale measure is indeed a probability measure. Section 4 discusses several hedging aspects while section 5 presents some simulation results. Proofs are postponed to section 7.

2 The Model

As outlined in the previous section our model is intended to combine three particular features. For this, assume that the stock price at $t = 0$ equals S_0. In the case that no jumps occur, we have, at time t,

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right),$$

following Black and Scholes (1973). Here μ is a drift, σ is the volatility and W is a Brownian Motion. At time $t = \tau_1$, the first jump of an associated Poisson Process N, we have

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right)(1 + U_1).$$

The "jump-size" U_1 describes abrupt changes in percentages of the former price. Therefore $U_1 > -1$. In theory U_1 may take on any positive value, but realistic values are less than $+1$. Finally, to incorporate "fade-away" effects we introduce a decay function h defined on the positive real line. For convenience, we set $h(t) = 0$ for $t < 0$. Then, if no further jumps occur, we have

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right)(1 + U_1 h(t - \tau_1)).$$

Typically, the function h is nonnegative and nonincreasing on the positive real line. A typical example is $h(x) = \exp[-cx]$, $x \geq 0$, $c \geq 0$. The choice of $c = 0$ leads to $h \equiv 1$, in which case we have no fade-away effects, i.e., $h \equiv 1$ corresponds to the pure jump-diffusion case. On the other hand, a large c leads to a fast downweighing of a jump.

Proceeding in this way, and denoting with $\tau_1 < \tau_2 < \ldots$ the successive jumps of the Poisson Process N, our final expression for the price becomes

$$S_t = S_0 \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right) \prod_{i=1}^{N_t} \left(1 + U_i h(t - \tau_i)\right). \quad (1)$$

Throughout this paper we shall make the following assumptions:

- W is a standard Brownian Motion
- N is a Poisson Process with constant intensity λ
- $U_1, U_2, \ldots$ are independent and identically distributed with finite second moment

- The function h vanishes for $t < 0$ and is continuously differentiable on $t \geq 0$.
- The processes W, N and the variables $(U_i)_i$ are independent of each other.

Let $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ be a filtered probability space carrying our random processes such that S is adapted to $(\mathcal{F}_t)_{t\geq 0}$.

Remark 2.1. Though, in our model (1), the volatility of the "continuous part" is still a constant, the overall volatility, caused by the jumps, is random. Furthermore, downweighing may and will lead to instationarities.

Typically, S will not be a Markov process. However, we have the following result:

Lemma 2.2. *Assume $h(x) = h(0)\exp(-cx)$ and set $J_t := \prod_{i=1}^{N_t}(1 + U_i h(t - \tau_i))$. Then the process $(W_t, J_t)_{t\geq 0}$ is a Markov process. If moreover $c = 0$, then S itself is Markovian.*

3 The Minimal Martingale Measure

In this section we determine the minimal martingale measure associated with the process (1). This measure, say $\hat{Q}$, will have a density L_T w.r.t. the original measure $\mathbb{P}$ driving the process S:

$$d\hat{Q} = L_T d\mathbb{P}. \tag{2}$$

Here, T is a finite horizon. In applications, T will be the maturity of a contingent claim associated with $(S_t)_{0\leq t\leq T}$. If H_T denotes the payoff at time T, then the present value of H_T equals

$$H_0 := e^{-rT}\hat{\mathbb{E}}(H_T),$$

where r is the market interest rate and $\hat{\mathbb{E}}$ denotes the expectation w.r.t. $\hat{Q}$.

Since our market model is incomplete, the martingale measure is not unique. The minimal martingale measure is the equivalent martingale measure $\hat{Q}$, such that any square-integrable P-martingale, orthogonal to S under P, is also orthogonal to S under $\hat{Q}$. This martingale measure is related to a hedging strategy, which minimizes the local risk inherent in the non-perfect hedge, the so-called locally risk-minimizing strategy. For a detailed exposition of the concepts in discrete time we refer to Föllmer and Schied (2002); for the continuous time case, Bingham and Kiesel (2004) give an accessible approach to pricing and hedging in incomplete markets.

In general, computation of L_T is not simple, in particular if the model for S is complicated.

Therefore we first consider S evaluated on an equidistant time grid $0 = t_0 < t_1 < t_2 < \ldots < t_q = T$, where $t_i = \frac{iT}{q}, 0 \leq i \leq q$. Put $\delta = \frac{T}{q}$. Following Dothan (1990), the density L_T (in discrete time) may then be computed as follows:

Set

$$r_j = \frac{S_{t_j}}{S_{t_{j-1}}} - 1, \qquad 1 \leq j \leq q$$

and let

$$r_j = M_j - M_{j-1} + \mu_j \equiv \Delta M_j + \mu_j$$

with

$$\mu_j = \mathbb{E}_{\mathbb{P}}\left(r_j | \mathcal{F}_{j-1}\right)$$

be the associated Doob-Meyer decomposition of $r_1, \ldots, r_q$. Here, $\mathcal{F}_j$ is the σ-field generated by the process up to time t_j. Then we have

$$S_j = S_0 \prod_{i=1}^{j} [1 + \mu_i + \Delta M_i].$$

The corresponding minimal density then equals

$$L_q \equiv L_T = \prod_{i=1}^{q} \left[1 - \frac{\mu_i \Delta M_i}{\mathbb{E}_{\mathbb{P}}(\Delta^2 M_i | \mathcal{F}_{i-1})} \right]. \tag{3}$$

Dothan (1990) derived L_T without discussing any minimality property. This was independently done by Schachermayer (1993).

In the following remark we show how the formulas for pricing and hedging under the minimal martingale measure can easily be derived in a one-period setting.

Remark 3.1. Consider a one-period setting, zero interest rate and write ΔS for $\Delta S_1 = S_1 - S_0$. In this remark, we simply write $\mathbb{E}$ for $\mathbb{E}_{\mathbb{P}}$. The aim is to price and hedge a contingent claim H which is due at time 1 where the claim can not be replicated perfectly. Therefore, at time 1, we may face a financial loss and we aim at minimizing $\mathbb{E}[(H - c - \xi \Delta S)^2]$ w.r.t. (c, ξ). c represents the price of the claim at time 0 and ξ is the hedge ratio, i.e., the number of shares to buy according to the chosen strategy. Note that $\Delta S = \mu_1 + \Delta M$. Setting the derivatives equal to zero yields on one side that $0 = \mathbb{E}(H - c^* - \xi^* \Delta S)$ and on the other side

$$0 = \mathbb{E}((H - c^* - \xi^* \Delta S)\Delta S) = \mathbb{E}((H - c^* - \xi^* \Delta S)\Delta M).$$

Thus we have two expressions for the optimal ξ,

$$\xi^* = \frac{\mathbb{E}\big[(H - c)\Delta S\big]}{\mathbb{E}((\Delta S)^2)} = \frac{\mathbb{E}\big[(H - c)\Delta M\big]}{\mathbb{E}((\Delta M)^2)}.$$

Observe that the first expression corresponds to (9) in the multi-period case. For the optimal c we have $c^* = \mathbb{E}(H - \xi^* \Delta S)$. After inserting our second expression for ξ^*,

$$c^* = \mathbb{E}(H) - \mu_1 \frac{\mathbb{E}(H \Delta M)}{\mathbb{E}((\Delta M)^2)} = \mathbb{E}\left(\left(1 - \frac{\mu_1 \Delta M}{\mathbb{E}((\Delta M)^2)} \right) H \right) = \hat{\mathbb{E}}(H),$$

where $\hat{\mathbb{E}}$ is the expectation under the minimal martingale measure; note that $(1 - \ldots)$ is equal to the density L_1 given in (3). The multi-period case considered in (9) is obtained by backward induction.

In Theorem 3.2 below we shall derive an explicit formula for L_q when S (properly discounted) follows (1). In Theorem 3.3 we present the continuous time version. This will be obtained from Theorem 3.2 as an almost sure limit upon letting $q \to \infty$. Note, however, that the discrete-time version is of interest in itself for Monte Carlo approximations of S and H_0.

To formulate our first result, recall the model components of (1). Set

$$P_{j-1} = \prod_{k=1}^{N_{t_{j-1}}} \frac{1 + U_k h(t_j - \tau_k)}{1 + U_k h(t_{j-1} - \tau_k)} \tag{4}$$

and

$$H_1(z) = \mathbb{E}(U_1) \int_0^z h(x)dx \qquad H_2(z) = \mathbb{E}(U_1^2) \int_0^z h^2(x)dx.$$

Theorem 3.2. *For the discounted model of (1), i.e., for $e^{-rt}S_t$, the minimal martingale density in discrete time equals*

$$L_q = \prod_{j=1}^{q}(1 - l_j),$$

with

$$\begin{aligned} l_j &= \frac{1 - P_{j-1}^{-1}\exp[(r-\mu)\delta - \lambda H_1(\delta)]}{1 - \exp[\delta\sigma^2 + \lambda H_2(\delta)]} \\ &\times \left\{1 - \exp\left[-\frac{\sigma^2}{2}\delta + \sigma\Delta W_j - \lambda H_1(\delta)\right] \prod_{k=N_{t_{j-1}+1}}^{N_{t_j}} [1 + U_k h(t_j - \tau_k)]\right\}. \end{aligned}$$

Theorem 3.3 presents the limit of L_q in continuous time.

Theorem 3.3. *In continuous time $0 \le t \le T$, the minimal density equals*

$$\begin{aligned} L_T &= \prod_{i=1}^{N_T}(1 - U_i h(0) I_1(\tau_i)) \exp\left\{\int_0^T \lambda h(0)\mathbb{E}(U_1) I_1(t)dt\right\} \\ &\times \exp\left\{-\int_0^T \frac{\sigma^2 I_1^2(t)}{2}dt - \int_0^T \sigma I_1(t)dW_t\right\}, \end{aligned}$$

where

$$I_1(t) = \frac{\mu - r + \lambda\mathbb{E}(U_1)h(0) + \sum_{m=1}^{N_{t-}} \frac{U_m h'(t-\tau_m)}{1+U_m h(t-\tau_m)}}{\sigma^2 + \lambda\mathbb{E}(U_1^2)h^2(0)}$$

and N_{t-} denotes the left-continuous version of N_t.

Remark 3.4. In a pure jump-diffusion process without shot-noise effects, the function h equals $h(0)$ on $t \ge 0$ so that $h' \equiv 0$ there. Hence in this case the function I_1 simplifies a lot and becomes

$$I_1(t) = \frac{\mu - r + \lambda\mathbb{E}(U_1)h(0)}{\sigma^2 + \lambda\mathbb{E}(U_1^2)h^2(0)} \quad \text{– a constant.} \tag{5}$$

Lamberton and Lapeyre (1997) studied the jump-diffusion process under the condition

$$\mu - r + \lambda\mathbb{E}U_1 h(0) = 0. \tag{6}$$

In this case, $I_1(t) \equiv 0$ and therefore $L_T = 1$, i.e., $e^{-rt}S_t$ is already a martingale under $\mathbb{P}$. If shot-noise effects are present, (6) is no longer sufficient to make $e^{-rt}S_t$ a martingale under $\mathbb{P}$. Note also that, under $h' = 0$ and with deterministic jumps, our formula for L_T reduces to the one obtained by Arai (2004) in the one-factor case. Finally, if there are no jumps at all, our model reduces to the Black-Scholes (1973) model with

$$I_1(t) \equiv \frac{\mu - r}{\sigma^2} \text{ and } \lambda = 0.$$

The density L_T becomes

$$L_T = \exp\left\{-\frac{(\mu - r)^2}{2\sigma^2}T - \frac{(\mu - r)}{\sigma}W_T\right\},$$

a well-known fact from the Black-Scholes world.

Since the form of L_T is complicated one may wonder if L_T defines a density of a proper distribution or rather a density of a signed measure. It is well known that the minimal martingale density may take on negative values if the underlying model admits jumps. In the following Propositions we give conditions, which ensure that the minimal martingale measure is necessarily a probability measure.

Proposition 3.5. *Assume that $h' \leq 0$ and $h \geq 0$. Then the density L_T is positive with probability 1 if either of the following conditions hold:*

(i) *$U_i \in [0, b)$ for all i and*

$$b \leq \frac{\sigma^2 + \lambda \mathbb{E}(U_1^2) h^2(0)}{h(0)[\mu - r + \lambda \mathbb{E}(U_1) h(0)]} \,. \tag{7}$$

(ii) *$U_i \in (-h(0)^{-1}, 0]$ for all i and*

$$\mu - r + \lambda \mathbb{E}(U_1) h(0) \geq 0 \,. \tag{8}$$

In the case when $h' \leq 0$ and at the same time positive as well as negative jumps are present, the density becomes negative with positive probability. It is clear, that in such a situation prices will allow for arbitrage. For example, assume that on the set $A \in \mathcal{F}_T$ the density is negative and A has a positive probability. Thus, the derivative paying 1 at T if A occurs has the price $\hat{E}(1_A) = \mathbb{E}(L_T 1_A) < 0$ and therefore generates an arbitrage possibility.

The next result discusses conditions under which the pure jump-diffusion process produces nonnegative L_T's.

Proposition 3.6. *Assume that $U_1 \in (a, b)$ with $-1 \leq a < 0 < b$ and $h' = 0$. If $\mu - r + \lambda \mathbb{E}(U_1) h(0) > 0$, then the density L_T is positive with probability 1 iff (7) holds. On the other side, for $\mu - r + \lambda \mathbb{E}(U_1) h(0) < 0$, this is the case iff*

$$a \geq \frac{\sigma^2 + \lambda \mathbb{E}(U_1^2) h^2(0)}{h(0)[\mu - r + \lambda \mathbb{E}(U_1) h(0)]}.$$

For $\mu - r + \lambda \mathbb{E}(U_1) h(0) = 0$ no additional condition is needed for positivity of L_T.

4 Hedging

The minimal martingale measure is the counterpart of a hedging strategy which minimizes the local quadratic risk. A short exposition of these concepts may be found in Prigent (2003, Chapter 2.3.3) while for a detailed analysis in discrete time we refer to Föllmer and Schied (2002). In the following we present the details under shot noise effects, in discrete time.

A trading strategy is given by a sequence $(\theta_k^0, \theta_k^1)_{k \in \{0,\dots,T\}}$. θ_k^0 denotes the amount of money invested in the risk-free bank account at time k. While θ_k^0 is a random variable adapted to the relevant filtration, the variable θ_k^1, denoting the number of shares held in period k, is assumed to be predictable.

Consider a European claim with maturity T, represented by a random variable H which is $\mathcal{F}_T$-measurable. Define $V_k := \hat{\mathbb{E}}(H|\mathcal{F}_k)$, the value of the claim at time k under the minimal martingale measure. Basically, the locally risk minimizing strategy in our setting is given by

$$\theta_k^1 = \frac{\mathbb{E}_{\mathbb{P}}\Big(\Delta V_k \, \Delta S_k \Big| \mathcal{F}_{k-1}\Big)}{\mathbb{E}_{\mathbb{P}}\big(\Delta S_k^2 | \mathcal{F}_{k-1}\big)} \tag{9}$$

and $\theta_k^0 = V_k - \theta_k^1 S_k$, where here and in the following we put $t_k = k$ for short. For clarification we also emphasized that the expectation is taken w.r.t. the objective measure $\mathbb{P}$.

Recall P_k from (4) and let μ_k be again defined through

$$1 + \mu_k = \mathbb{E}_{\mathbb{P}}\left(\frac{S_k}{S_{k-1}}\middle|\mathcal{F}_{k-1}\right).$$

See (11) below for more details about μ_k under shot noise. Using the structure of our model, we obtain the following representation.

Theorem 4.1. *The hedge ratio for the locally risk-minimizing strategy equals*

$$\theta_k^1 = \frac{\mathbb{E}_{\mathbb{P}}\big[V_k B_k|\mathcal{F}_{k-1}\big] - \mu_k V_{k-1}}{S_{k-1}\left[(1+\mu_k)^2 e^{\delta\sigma^2 + \lambda H_2(\delta)} - 1 - 2\mu_k\right]},$$

with

$$B_k := P_{k-1}\exp\left[(\mu - \frac{\sigma^2}{2})\delta + \sigma\Delta W_k\right] \prod_{j=N_{t_{k-1}}+1}^{N_{t_k}} (1 + U_j \cdot h(t_k - \tau_j)) - 1.$$

5 Simulation Results

This section illustrates the proposed model in terms of a small simulation study and gives a detailed comparison of the resulting hedging strategy with the delta-hedging in the Black-Scholes world.

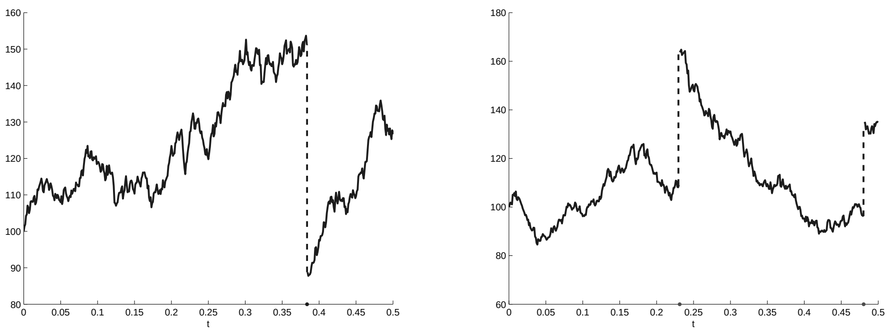

Figure 1: Two paths of the processes as defined in (1). For demonstration purposes, the distribution of the jump size was chosen so that typically large jumps occur; in the left plot we allow for downward jumps and in the right for upward jumps. The decay function equals $h(x) = e^{-4x}$. After the jump a strong return to the pre-jump level can be observed.

Path properties. Following Proposition 3.5 we will consider the cases where $\hat{Q}$ is a probability measure, i.e., we consider the case where jumps occur only in one direction. For practical purposes the case where only negative jumps occur is the most important one; in the simulations we typically concentrate on this case. To show the advantages of the minimal martingale hedging, we chose the jump size density so as to produce quite big jumps; for the negative case, the density of the U's was

$$f_-(u) = \frac{1}{0.3} 1_{[-0.5,-0.2]}(u)$$

and the reflected density for the case of only positive jumps. Further parameters are: $\mu = 0.8, r = 0, \sigma = 0.4$ and the jump intensity $\lambda = 2$, i.e., we expect two jumps in a year. For the decay function we put $h(x) = e^{-4x}$. Hence the shot-noise effect fades away quite rapidly. The hedging strategy taking this into account will clearly be able to take advantage of this. Figure 1 shows two typical paths of a jump-diffusion with shot-noise effects, i.e., the model as defined in (1) with parameters given as above.

Smile effects. Not surprisingly, the multiplicative shot-noise model is able to reproduce so-called smile effects. Smile effects account for the underrepresentation of large downward jumps in the Black-Scholes model. To study this, we compute implied volatilities from the option prices obtained via the minimal martingale measure. Figure 2 shows the resulting graph with implied volatilities for maturities up to half a year and for several strikes; the case where only negative jumps occur was considered. The graph matches the typical structure of volatility surfaces implied by real data. Compare, for example, Figure 13.8 in Cont and Tankov (2004).

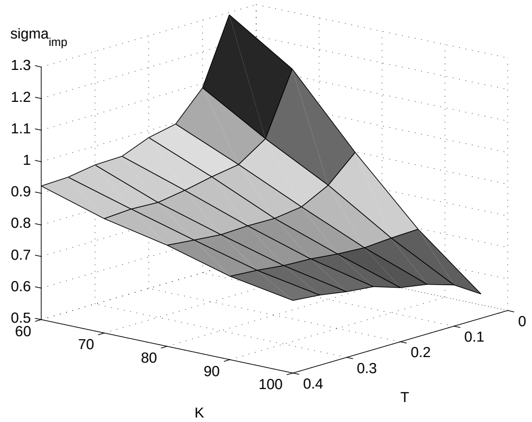

Figure 2: The implied volatility surface obtained from option prices under the minimal martingale measure.

Influence of the shot-noise effect. To study the impact of the shot-noise effect we consider the case where, as before, we have only downward jumps, $h(x) = e^{-cx}$ with varying c. First, the impact on the distribution of the stock price under the objective measure is the following: if $c = 0$ an occuring jump persists, while if c is large, the effect of the downward jump will fade away rapidly and the stock price fastly reverts to the pre-jump level. Second, we study the effect of c on the distribution of S under the minimal martingale measure, which is used for pricing. As prices are martingales under $\hat{Q}$, the above discussed effect under the objective measure will be compensated in a certain way. More precisely, consider $t = 0$ and a time horizon of T =1 month and the following two scenarios:

S1 There was no jump at $t = 0$

S2 There was a downward jump at $t = 0$

Figure 4 shows the density of S_T under $\hat{Q}$ in both cases. The hump on the left is due to the jumps. Consider **S1**, i.e., the left plot in Figure 4. With increasing c, the hump gets smoothed away, as under $\mathbb{P}$. On the other side, a second (smaller) hump appears on the very

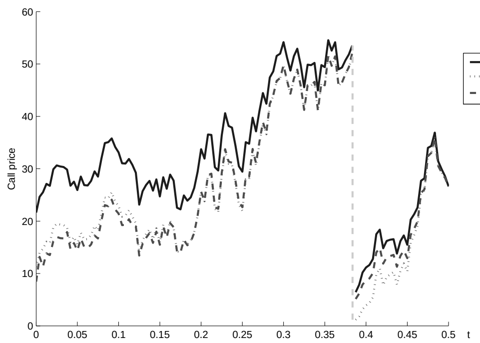

Figure 3: Call price with strike $K = 100$ and Maturity $T = 0.5$ computed according to the MM (solid line), Black-Scholes (dash-dotted line) and Black-Scholes with estimated σ (dotted line).

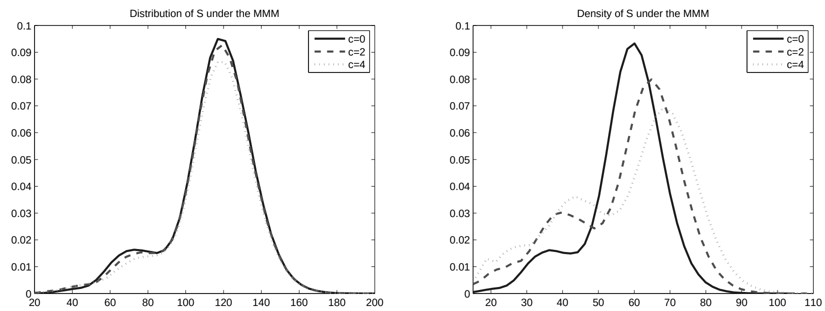

Figure 4: Distribution of S_T under the MMM $\hat{Q}$ for different choices of c $(0, 0.2$ and $0.4)$. Left: scenario **S1**. Right: scenario **S2** (See text for details).

left, which shows that the distribution is shifted to the left. The option prices in Figure 5 perfectly reflect this, as call prices decline with increasing c.

Next, consider **S2**, i.e., the right plot in Figure 4. This time, the distribution is strongly shifted to the right, especially the center of the large peak moves strongly. Simultaneously, the hump caused by jumps becomes clearer. Recall, that in this scenario at $t = 0$ a downward jump occurred and the shot-noise part leads to a faster reversion to the pre-jump level the higher c is. Note also, that as in S1, a small hump on the left occurs. However, the shift in the mean is dominating this scenario, which explains the increasing call prices in Figure 5.

One further aspect of the impact of c is studied at the end of the next paragraph, where the quality of the hedging strategy for different levels of c is examined.

Hedging. This paragraph analyses the hedging strategy according to Theorem 4.1 through simulations. For this, we compare four hedging strategies:

BS The delta-hedging strategy of Black Scholes, simply neglecting the possibility of jumps

BS$_i$ The delta-hedging strategy of Black Scholes, this time using the volatility implied in option prices. The option prices are computed under the minimal martingale measure

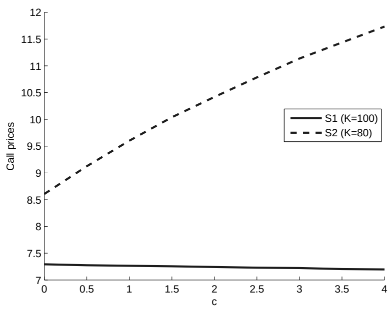

Figure 5: Dependence of option prices on c. We show Call prices in scenarios $\mathbf{S1}$ and $\mathbf{S2}$ for different c. Parameters in $\mathbf{S1}$: $K = 100, S_0 = 100, T = 1/12$, in $\mathbf{S2}$: $K = 80, S_{0-} = 125.9, S_0 = 83.9, T = 1/12$.

BS$_e$ The delta-hedging strategy of Black Scholes when the volatility is estimated from the data using the sample variance

MM The locally risk-minimizing hedging strategy given by Theorem 4.1

We implemented the above hedging strategies and applied them to the left path shown in Figure 1. The results of the simulation are plotted in Figure 6. On the l.h.s. we show weekly hedging over a period of half a year while the r.h.s. shows daily hedging for the last 27 days.

Comparing the paths reveals the following: first, the strategies BS and BS$_e$ most heavily react on the jump. Before the occurrence of the jump the strategies are quite close, because the volatility estimate is close to the underlying volatility. After the jump, however, the estimated volatility increases and the effect of the downward jump remains present in BS$_e$ until maturity. Second, the strategy MM is the most stable one. Moreover, because only this strategy incorporates the noise effect correctly, it is able to adjust for this and shows the smallest reaction on the downward jump. As we consider only down-jumps, the call prices under this strategy are also cheaper then in the no-jump case. Finally, it seems remarkable that the strategy using implied volatility is closest to the strategy MM.

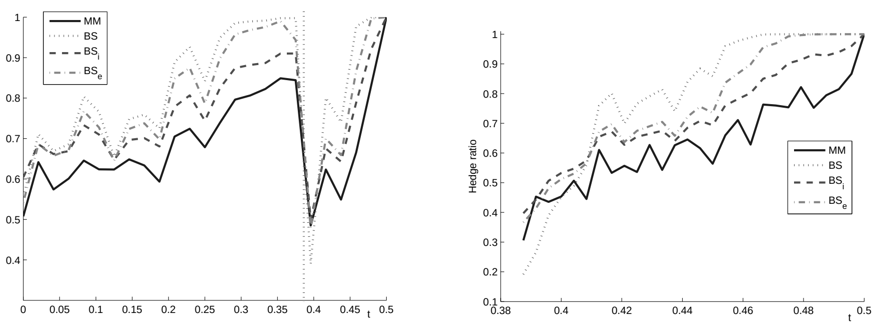

Figure 6: The hedging strategies BS and MM, according to the l.h.s. path in Figure 3. We plot the following hedging strategies (refer to text for details): BS (dotted), BS$_e$ (dash-dotted), BS$_i$ (dashed) and MM (solid). Left: weekly hedging. Right: daily hedging for the last twenty time points.

As the considered market is incomplete, the hedging strategies have a remaining risk. It is therefore interesting to analyse the variance of the suggested hedging strategies. Moreover, we want to analyse more quantitatively how incorporating the shot-noise effect can improve the hedging performance. For this, we simulated the profit and loss (P&L) of the above hedging strategies and computed the resulting variances. The results of 10^6 simulations are shown in Figure 7. On the l.h.s. we show the direct P&L of a static hedge over the period of a month. For the r.h.s. we fixed a 1-month history with a downward jump in the middle. The resulting variances are given in Table 1.

The results show the following: First, the strategy MM has the lowest variance in all cases - as expected. This should hold true by definition. In the plot on the l.h.s. BS and BS_e almost coincide. Note that besides the differences in variances they also differ in mean. This is true, because the first two methods neglect the possibility of future jumps. Second, examining the plot on the r.h.s. as well as the results given in Table 1 clearly shows that incorporating the noise effect is advantageous in terms of the variance of the P&L. However, the means of the strategy BS_e are positive, which is a desirable property. This is due to the particular example chosen. Recall that a Call is considered and a downward jump occurred in the past and, by the noise effect, is expected to be annihilated in the near future. This can of course not be extended to other scenarios.

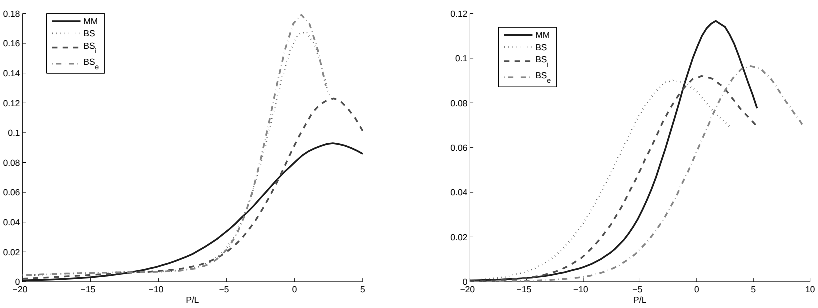

Figure 7: The profit and loss distribution of a static one-month hedge according to BS (dotted), BS_e (dash-dotted), BS_i (dashed) and MM (solid). Left: directly starting from time 0. Right: using a path history of 1 month, including a jump.

Table 1 also allows to compare the pure jump-diffusion case ($c = 0$) with the shot-noise case ($c = 4$). We basically concentrate on the comparison with the strategy BS. The table gives means and variances of the P&L of the implemented strategies under the scenarios S1 (no jump in the past) and S2 (jump at $t = 0$) introduced on page 8. In scenario S1, observe that the difference in means is less clear when $c = 4$. This is because the shot-noise effect compensates the downward jumps to a certain extent, from which the strategy BS takes advantage of. In the case where a jump occurred just before (scenario S2), the situation changes. Here, not incorporating the strong upward trend is fatal and BS shows a large negative mean of about -6 if $c = 4$. In all cases, the strategy MM has of course mean zero. For the variances, we give the ratios of the variances of BS over MM: Clearly, the P&L of strategy MM always has a lower variance. However, it can in particular take advantage of the special features of the path behavior in the case where c is large and just before a jump occurred. This is because this strategy anticipates the shot-noise effect correctly and therefore a much smaller variance in the P&L can be obtained.

Summarizing, the above findings show that pricing according to the minimal martingale measure gives reasonable prices for European Calls. Moreover, the hedging strategy suggested in Theorem 4.1 is superior to other hedging strategies in terms of more stable hedges

Table 1: Estimated mean and variance of the tracking error resulting from the hedging strategies according to the simulations in Figure 7.

	S1		S2	
Strategy	mean	variance	mean	variance
		c=0		
BS	-3.18	67.24	-1.58	15.30
BS_i	0.13	49.27	0.07	15.53
BS_e	-2.92	63.81	7.50	17.90
MM	0	40.34	0	12.93
		c=4		
BS	-2.44	44.61	-6.26	21.11
BS_i	0.17	35.20	-4.56	20.52
BS_e	-2.18	42.51	3.82	15.95
MM	0	32.35	0	8.03

Table 2: The scenarios introduced on page 8 were used (**S1** – directly starting at $t = 0$; **S2** – a jump occurred at $t = 0$). The shot-noise effect was $h(x) = \exp(-cx)$ and parameter c was varied.

	S1	S2
c=0	1.67	1.18
c=4	1.38	2.62

and lower variance. Especially, if a jump occurred and an investor expects a shot-noise effect, this hedging strategy shows excellent performance.

6 Conclusions

This paper proposes a new model for asset prices which incorporates shot-noise effects. The minimal martingale measure is derived in discrete and continuous time and the corresponding hedging strategies are given. In this model the minimal martingale measure is typically a signed measure, and we provide conditions which are sufficient to guarantee that the minimal martingale measure is a probability measure. Furthermore, the obtained option prices feature smile effects. We also analyse the consequences for pricing and hedging in a simulation study and find out that its hedging performance is superior to other models.

7 Appendix

First, we prove Lemma 2.2 on Markovianity of S.

Proof of Lemma 2.2. We will make use of the general results on Markovianity of solutions of stochastic differential equations, as provided in Protter (2004), Section V.6. Set $\tilde{J}_t = \sum_{i=1}^{N_t} U_i$. First, observe that $\tilde{J}$ has stationary and independent increments, hence is a Lévy process. Second, as $h(x) = h(0)\exp(-cx)$, J is the solution of

$$dJ_t = -cJ_{t-}\,dt + J_{t-}\,d\tilde{J}_t.$$

Then, Theorem V.34 in Protter (2004) yields that J is a Markov process. As W is itself a Markov process and is independent of J we obtain that (W, J) is a Markov process. In the case where $c = 0$ we have that S is the solution of the following SDE:

$$dS_t = S_{t-}\left(\mu dt + \sigma dW_t + dJ_t\right).$$

As $\sigma W + J$ is a Lévy process, S itself is a Markov process in this case. □

We continue with a small lemma and a corollary which are necessary for the proofs following thereafter.

Lemma 7.1. *For $n \in \mathbb{N}$ we have*

$$\mathbb{E}\left[\prod_{k=1}^{N_{t_j}-N_{t_{j-1}}}\Big(1 + U_{N_{t_{j-1}+k}} \cdot h(t_j - \tau_{N_{t_{j-1}+k}})\Big)^n \,\Big|\, \mathcal{F}_{t_{j-1}}\right]$$
$$= \exp\left\{\lambda\frac{T}{q}\left[\mathbb{E}\left((1 + U_1 \cdot h(\eta))^n\right) - 1\right]\right\}, \tag{10}$$

where η is independent of U_1 and uniformly distributed over $[0, \delta]$.

Proof. The left-hand side of (10) equals

$$\sum_{l,m\geq 0} 1_{\{N_{t_{j-1}}=l\}}\mathbb{E}\left[1_{\{N_{t_j}-N_{t_{j-1}}=m\}}\prod_{k=1}^{m}\left(1 + U_{l+k}\cdot h(t_j - \tau_{l+k})\right)^n \,\Big|\, \mathcal{F}_{t_{j-1}}\right].$$

For given l and m, the variables $\tau_{l+1} \leq \ldots \leq \tau_{l+m}$ are distributed like m order statistics from a sample of independent variables with uniform distribution on $[t_{j-1}, t_j]$. Since the U's are independent of the underlying Poisson Process it follows that the last expectation equals

$$e^{-\lambda\frac{T}{q}}\frac{\left(\lambda\frac{T}{q}\right)^m}{m!}\mathbb{E}^m\left[(1 + U_1 h(\eta))^n\right].$$

Summation over all l and m yields the result. □

Lemma 7.2. *With n, η as above and $\sigma > 0$ we have*

$$\mathbb{E}\left[e^{\sigma(W_{t_j}-W_{t_{j-1}})}\prod_{k=1}^{N_{t_j}-N_{t_{j-1}}}\Big(1 + U_{N_{t_{j-1}+k}} \cdot h(t_j - \tau_{N_{t_{j-1}+k}})\Big)^n \,\Big|\, \mathcal{F}_{t_{j-1}}\right]$$
$$= \exp\left\{\frac{T}{q}\left(\frac{\sigma^2}{2} + \lambda\left[\mathbb{E}(1 + U_1 \cdot h(\eta))^n - 1\right]\right)\right\}.$$

Proof. Applying Lemma 7.1 and using the independence of W and N, $U_1, U_2, \ldots$, we get the desired result. □

Proof of Theorem 3.2. We may assume w.l.o.g. that $r = 0$. Otherwise, replace μ with $\mu - r$. We first compute

$$\mu_{t_j} = \mathbb{E}\left(\frac{S_{t_j}}{S_{t_{j-1}}}\,\Big|\,\mathcal{F}_{t_{j-1}}\right) - 1.$$

The conditional expectation equals

$$\begin{aligned}
&\mathbb{E}\left(\frac{S_{t_j}}{S_{t_{j-1}}}\,\Big|\,\mathcal{F}_{t_{j-1}}\right)\\
&= \mathbb{E}\left[e^{\left(\mu-\frac{\sigma^2}{2}\right)\delta+\sigma\Delta W_j}\cdot P_{j-1}\cdot\prod_{k=N_{t_{j-1}}+1}^{N_{t_j}}(1+U_k\cdot h(t_j-\tau_k))\,\Big|\,\mathcal{F}_{t_{j-1}}\right].
\end{aligned}$$

From Lemma 7.2 it follows with $n = 1$ there, that

$$\begin{aligned}
\mu_j &= \mathbb{E}\left(\frac{S_{t_j}}{S_{t_{j-1}}}\,\Big|\,\mathcal{F}_{t_{j-1}}\right) - 1 = e^{\left(\mu-\frac{\sigma^2}{2}\right)\delta}P_{j-1}\cdot e^{\delta\left(\frac{\sigma^2}{2}+\lambda\mathbb{E}(U_1\cdot h(\eta))\right)} - 1\\
&= P_{j-1}\exp\left[\mu\delta+\lambda\mathbb{E}(U_1)\int_0^\delta h(x)dx\right] - 1, \qquad (11)
\end{aligned}$$

where η is independent of U_1 and uniformly distributed over $[0,\delta]$. The next step will be to compute ΔM_j. Now,

$$\begin{aligned}
\Delta M_j &= \frac{S_{t_j}}{S_{t_{j-1}}} - \mathbb{E}\left(\frac{S_{t_j}}{S_{t_{j-1}}}\,\Big|\,\mathcal{F}_{t_{j-1}}\right)\\
&= e^{\mu\delta}P_{j-1}\left[e^{-\delta\frac{\sigma^2}{2}+\sigma\Delta W_j}\prod_{k=N_{t_{j-1}}+1}^{N_{t_j}}(1+U_k\cdot h(t_j-\tau_k)) - e^{\lambda H_1(\delta)}\right]\\
&=: e^{\mu\delta}P_{j-1}\left[A_j - B\right]. \qquad (12)
\end{aligned}$$

Since $B = \mathbb{E}(A_j|\mathcal{F}_{t_{j-1}})$, the predictable quadratic variation takes on the form

$$\mathbb{E}\left(\Delta^2 M_j|\mathcal{F}_{t_{j-1}}\right) = e^{2\mu\delta}P_{j-1}^2\left[\mathbb{E}(A_j^2|\mathcal{F}_{t_{j-1}}) - B^2\right].$$

Equation (10) with $n = 2$ yields

$$\begin{aligned}
\mathbb{E}(A_j^2|\mathcal{F}_{t_{j-1}}) &= e^{-\delta\sigma^2}\mathbb{E}\left[e^{2\sigma\Delta W_j}\prod_{k=N_{t_{j-1}}+1}^{N_{t_j}}(1+U_k\cdot h(t_j-\tau_k))^2\,\Big|\,\mathcal{F}_{t_{j-1}}\right]\\
&= \exp\left\{\delta\left(\sigma^2+\lambda\left[\mathbb{E}\left((1+U_1\cdot h(\eta))^2\right)-1\right]\right)\right\}\\
&= \exp\left\{\delta\sigma^2+2\lambda H_1(\delta)+\lambda\mathbb{E}(U_1^2)\int_0^\delta h^2(x)dx\right\}.
\end{aligned}$$

We may therefore conclude

$$\begin{aligned}
\mathbb{E}\left(\Delta^2 M_j|\mathcal{F}_{t_{j-1}}\right) &= \exp\left[2(\mu\delta+\lambda H_1(\delta))\right]\\
&\quad\cdot P_{j-1}^2\cdot\left[\exp\left(\delta\sigma^2+\lambda H_2(\delta)\right)-1\right]. \qquad (13)
\end{aligned}$$

Plugging (11) – (13) into (3) yields the assertion of Theorem 3.2. □

Proof of Theorem 3.3. Assume that q has been chosen so large that for each j there is, if any, at most one jump in $(t_{j-1}, t_j]$. Note that q depends on the sample path of N and is therefore random. We may rewrite L_q from (3) as

$$L_q = \exp\left[\sum_{j=1}^{q} \ln(1-l_j) 1_{\{\tau_{N_{t_{j-1}}+1} \le t_j\}}\right] \exp\left[\sum_{j=1}^{q} \ln(1-l_j) 1_{\{\tau_{N_{t_{j-1}}+1} > t_j\}}\right].$$

Note that $\tau_{N_{t_{j-1}}+1} \le t_j$ if and only if $(t_{j-1}, t_j]$ contains exactly one jump. Hence the first factor in L_q takes into account all intervals which contain jumps while the second factor covers the empty intervals. If an interval is nonempty,

$$\prod_{k=N_{t_{j-1}}+1}^{N_{t_j}} (1 + U_k h(t_j - \tau_k)) = 1 + U_{N_{t_j}} h(t_j - \tau_{N_{t_j}}).$$

Conclude that

$$\begin{aligned}
&\sum_{j=1}^{q} \ln(1-l_j) 1_{\{\tau_{N_{t_{j-1}}+1} \le t_j\}} \\
&= \sum_{i=1}^{N_T} \ln\Bigg\{1 - \frac{1 - \prod_{m=1}^{i-1} \frac{1+U_m H(\tilde{t}_i - \delta - \tau_m)}{1+U_m H(\tilde{t}_i - \tau_m)} \exp((r-\mu)\delta - \lambda H_1(\delta))}{1 - \exp(\delta\sigma^2 + \lambda H_2(\delta))} \\
&\qquad \cdot \left[1 - \big(1 + U_i h(\tilde{t}_i - \tau_i)\big) \exp\Big(-\frac{\sigma^2}{2}\delta + \sigma(W_{\tilde{t}_i} - W_{\tilde{t}_i - \delta}) - \lambda H_1(\delta)\Big)\right]\Bigg\}.
\end{aligned}$$

Here $\tilde{t}_i$ is the right neighbor of τ_i on the grid. Now, letting q tend to infinity, we obviously have $\delta \to 0$ so that for $1 \le i \le N_T < \infty$:

$$\begin{aligned}
&\big(1 + U_i h(\tilde{t}_i - \tau_i)\big) \exp\left[-\frac{\sigma^2}{2}\delta + \sigma\left(W_{\tilde{t}_i} - W_{\tilde{t}_i - \delta}\right) - \lambda H_1(\delta)\right] \\
&\qquad \longrightarrow 1 + U_i h(0) \quad \text{with probability one.}
\end{aligned}$$

Finally, we apply l'Hospital's rule to get, as $\delta \to 0$:

$$\begin{aligned}
&\frac{1 - \prod_{m=1}^{i-1} \frac{1+U_m h(\tilde{t}_i - \delta - \tau_m)}{1+U_m h(\tilde{t}_i - \tau_m)} \exp((r-\mu)\delta - \lambda H_1(\delta))}{1 - \exp(\delta\sigma^2 + \lambda H_2(\delta))} \\
&\to \quad \frac{-\frac{\sum_{j=1}^{i-1} \prod_{m=1, m\ne j}^{i-1} (1+U_m h(\tau_i - \tau_m)) U_j h'(\tau_i - \tau_j)(-1)}{\prod_{m=1}^{i-1}(1+U_m h(\tau_i - \tau_m))} - [(r-\mu) - \lambda \mathbb{E} U_1 h(0)]}{-(\sigma^2 + \lambda \mathbb{E} U_1^2 h^2(0))} \\
&= \quad \frac{r - \mu - \lambda \mathbb{E} U_1 h(0) - \sum_{j=1}^{i-1} \frac{U_j h'(\tau_i - \tau_j)}{1+U_j h(\tau_i - \tau_j)}}{\sigma^2 + \lambda \mathbb{E} U_1^2 h^2(0)}.
\end{aligned}$$

Summarizing, we get with probability one

$$\exp\left[\sum_{j=1}^{q} \ln(1-l_j) 1_{\{\tau_{N_{t_{j-1}}+1} \le t_j\}}\right] \to \prod_{i=1}^{N_T} \left[1 - U_i h(0) I_1(\tau_i)\right]. \tag{14}$$

Next we study the contribution of the empty intervals. For each $1 \le j \le q$, define $\tilde{l}_j$ as l_j, with $U_{N_{t_{j-1}}+1}$ replaced by $\tilde{U}_{N_{t_{j-1}}+1} \equiv 0$. Then l_j and $\tilde{l}_j$ coincide on empty intervals. Hence

$$\begin{aligned}\sum_{j=1}^{q} \ln(1-l_j) 1_{\{\tau_{N_{t_{j-1}}+1} > t_j\}} &= \sum_{j=1}^{q} \ln(1-\tilde{l}_j) 1_{\{\tau_{N_{t_{j-1}}+1} > t_j\}} \\ &= \sum_{j=1}^{q} \ln(1-\tilde{l}_j) - \sum_{j=1}^{q} \ln(1-\tilde{l}_j) 1_{\{\tau_{N_{t_{j-1}}+1} \le t_j\}}.\end{aligned}$$

Applying (14) to the last sum with $\tilde{U}_i = 0$ in place of U_i, we obtain

$$\sum_{j=1}^{q} \ln(1-\tilde{l}_j) 1_{\{N_{t_{j-1}+1} \le t_j\}} \to 0 \text{ with probability one.} \tag{15}$$

To study $\sum_{j=1}^{q} \ln(1-\tilde{l}_j)$, we use the expansions

$$\ln(1-\tilde{l}_j) = -\Big[\tilde{l}_j + \frac{\tilde{l}_j^2}{2} + \frac{\tilde{l}_j^3}{3} + \dots\Big]. \tag{16}$$

Recall

$$\begin{aligned}\tilde{l}_j &= \frac{1 - \prod_{k=1}^{N_{t_{j-1}}} \frac{1+U_k h(t_{j-1}-\tau_k)}{1+U_k h(t_j-\tau_k)} \exp((r-\mu)\delta - \lambda H_1(\delta))}{1-\exp(\delta\sigma^2 + \lambda H_2(\delta))} \\ &\quad \cdot \Big[1 - \exp\Big(-\frac{\sigma^2}{2}\delta + \sigma(W_{t_j} - W_{t_j-\delta}) - \lambda H_1(\delta)\Big)\Big].\end{aligned}$$

As in the first part of this proof, the first ratio equals

$$\frac{r-\mu-\lambda \mathbb{E} U_1 h(0) - \sum_{k=1}^{N_{t_{j-1}}} \frac{U_k h'(t_j-\tau_k)}{1+U_k h(t_j-\tau_k)}}{\sigma^2 + \lambda \mathbb{E} U_1^2 h^2(0)} + o_{\mathbb{P}}(1) = -I_1(t_j) + o_{\mathbb{P}}(1).$$

The factor in brackets may be approximated by

$$\frac{\sigma^2}{2}\delta - \sigma(W_{t_j} - W_{t_{j-\delta}}) + \lambda H_1(\delta) - \frac{\sigma^2}{2}(W_{t_j} - W_{t_j-\delta})^2 + O\Big((\delta \ln \delta^{-1})^{3/2}\Big)$$

uniformly in $1 \le j \le q$, where higher powers of ΔW_j are bounded by the Lévy modulus of continuity. From this it follows like in the derivation of Îto's formula, that

$$\lim_{q\to\infty} \sum_{j=1}^{q} \tilde{l}_j = \sigma \int_0^T I_1(t) dW_t - \lambda \mathbb{E} U_1 h(0) \int_0^T I_1(t) dt. \tag{17}$$

Similarly,

$$\lim_{q\to\infty} \sum_{j=1}^{q} \tilde{l}_j^2 = \sigma^2 \int_0^T I_1^2(t) dt. \tag{18}$$

Higher order powers of $\tilde{l}_j$ are negligible. The assertion of the Theorem now follows from (14) – (18). □

Proof of Proposition 3.5. From the representation of L_T in Theorem 3.3 we find that L_T is nonnegative if and only if

$$1 \geq U_i h(0) I_1(\tau_i) \text{ for all } i \in \mathbb{N} \tag{19}$$

and all possible values of U_i and τ_i.

We first proof (i). Note that under $U_i \geq 0$ and $h' \leq 0$ the function I_1 is less than or equal to the I_1 corresponding to $h' \equiv 0$. Consequently, $U_i < b$ implies $U_i h(0) I_1(\tau_i) < 1$ and the assertion follows.

Second, for (ii), observe that $U_m h'(t-\tau_m) \geq 0$. Then, as $h(\cdot) \in [0,1]$, we obtain $1+U_m h(t-\tau_m) > 0$ and consequently $\frac{U_m h'(t-\tau_m)}{1+U_m h(t-\tau_m)} \geq 0$. Together with $\mu - r + \lambda \mathbb{E}(U_1) h(0) \geq 0$ we obtain $I_1(t) \geq 0$. Note that with $U_i \leq 0$, $I_1(t) \geq 0$ and $h(0) \geq 0$ imply $U_i h(0) I_1(\tau_i) \leq 0 < 1$ and so we conclude that the density is positive. □

Proof of Proposition 3.6. As in the previous proof we use condition (19). When $h' = 0$ on the positive real line,

$$I_1(\tau_i) = \frac{\mu - r + \lambda \mathbb{E}(U_1) h(0)}{\sigma^2 + \lambda \mathbb{E}(U_1^2) h^2(0)},$$

a constant. The assertion now follows from $a < U_i < b$. □

Proof of Theorem 4.1. Recall the decomposition $S_k = S_{k-1}\big(1 + \mu_k + \Delta M_k\big)$, where μ_k was $\mathcal{F}_{k-1}$-measurable and ΔM_k had conditional expectation zero. We find that $\Delta S_k = S_{k-1}(\mu_k + \Delta M_k)$. Therefore $\mathbb{E}(\Delta S_k^2 | \mathcal{F}_{k-1}) = S_{k-1}^2 [\mu_k^2 + \mathbb{E}((\Delta M_k)^2 | \mathcal{F}_{k-1})]$. The last expectation was calculated in (13).

As to the numerator, we consider $\Delta V_k \Delta S_k$ more closely. First,

$$\begin{aligned}\mathbb{E}\big(\Delta V_k \Delta S_k | \mathcal{F}_{k-1}\big) &= \mathbb{E}\big(V_k \Delta S_k | \mathcal{F}_{k-1}\big) - V_{k-1} \mathbb{E}\big(\Delta S_k | \mathcal{F}_{k-1}\big) \\ &= S_{k-1}\big[\mu_k \mathbb{E}(V_k | \mathcal{F}_{k-1}) + \mathbb{E}(V_k \Delta M_k | \mathcal{F}_{k-1}) - V_{k-1}\mu_k\big]\end{aligned}$$

From Equation (12) we know that $\mu_k + \Delta M_k = A_k e^{\mu\delta} P_{k-1} - 1$ so that altogether we get the following form for the hedging strategy:

$$\theta_k^1 = \frac{\mathbb{E}\Big[V_k\big(A_k e^{\mu\delta} P_{k-1} - 1\big) | \mathcal{F}_{k-1}\Big] - \mu_k V_{k-1}}{S_{k-1}\left[(\mu_k + 1)^2 \exp(\delta\sigma^2 + \lambda H_2(\delta)) - 2\mu_k - 1\right]},$$

which coincides with the one given in the Theorem. □

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