Differentiability of Point Process Models and Asymptotic Efficiency of Differentiable Functionals

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Abstract

In this paper we consider some different techniques allowing to construct asymptotically efficient estimators in point process models. In particular we establish L^2-differentiability for point processes with multiplicative intensities and thus can apply Hadamard differentiability techniques for the i.i.d. case. In the second part of the paper we extend some properties of differentiable functionals known from the i.i.d. situation to general LAN models of point processes. We establish the LAN condition for point processes with differentiable intensities and as consequence obtain optimality of various estimators in general intensity models.

1 Introduction

Let for $n \in \mathbb{N}$, $\mathcal{P}_n = \{P_{n,\alpha};\ \alpha \in I\}$ be a class of probability measures on $(\Omega_n, \mathcal{F}_n)$, where $\mathcal{F}_n = (\mathcal{F}_{nt})_{t\in[0,1]}$ is a filtration generated by point processes $N_n = (N_{n,t})_{t\in[0,1]}$. Let N_n have $\mathcal{F}_n$-predictable intensities $\lambda_{n,\alpha}$ w.r.t. $P_{n,\alpha}$, $\alpha \in I$, i.e. N_n has the Doob-Meyer decomposition

$$N_{n,t} = M_{n,\alpha}(t) + \int_0^t \lambda_{n,\alpha}(s)ds, \tag{1.1}$$

$M_{n,\alpha}$ a $P_{n,\alpha}$-martingale and $A_{n,\alpha}(t) = \int_0^t \lambda_{n,\alpha}(s)ds$ the compensator. The likelihood ratio on $\mathcal{F}_{n,t}$ under suitable continuity conditions is given by

$$\frac{dP_{n,\alpha_1}}{dP_{n,\alpha_o}}|_{\mathcal{F}_{n,t}} = \exp(\int_0^t \log \frac{\lambda_{n,\alpha_1}}{\lambda_{n,\alpha_o}} dN_n - \int_0^t (\lambda_{n,\alpha_1}(s) - \lambda_{n,\alpha_o}(s))ds) \tag{1.2}$$

for $\alpha_o, \alpha_1 \in I$ (cf. Jacod (1975), Liptser, Shiryaev (1978) and Karr (1985)). For general reference on point processes we refer to Bremaud (1981), Karr (1985) and Kutoyants (1984). Aalen (1978) introduced models with multiplicative intensities. Here I is assumed to be a subset of the caglad functions $\alpha : [0,1] \to [0,\infty)$ (caglad = left continuous, right hand limits) and $\lambda_{n,\alpha} = \alpha\lambda_n$, where λ_n is a basic predictable intensity and α is the (unknown) parameter. In iid models of point processes we have n observed independent copies $(N^{(1)}, \lambda^{(1)}), \ldots, (N^{(n)}, \lambda^{(n)})$ of the basic point process (N, λ). $N_n = \sum_{i=1}^n N^{(i)}$ is a sufficient statistic with intensity $\lambda_{n,\alpha} = \alpha\lambda_n = \alpha \sum_{i=1}^n \lambda^{(i)}$ in the multiplicative model, $\lambda_{n,\alpha} = \sum_{i=1}^n \lambda_\alpha^{(i)}$ in the general case.

The problem of estimation of the compensator $A_{n,\alpha}$ (or the intensities α) has been considered by Aalen (1978), Rebolledo (1978), Liptser and Shiryayev (1978). For point processes on $[0, t]$, $t \to \infty$ cf. Kutoyants (1984); for point processes on a fixed interval $[0, 1]$ we refer to Greenwood and Wefelmeyer (1989) and Millar (1990), while for the case of Poisson processes cf. Karr (1985), Rüschendorf (1989), Liese (1990), Kutoyants and Liese (1990). The aim of the present paper is twofold. On one hand side we extend the relationship between differentiability and efficiency to the estimation of functionals with values in general topological vector spaces in the general LAN case. For the iid case cf. van der Vaart (1988) and Gill (1986), for the estimation of real functionals cf. Pfanzagl and Wefelmeyer (1988) in the LAN case. We apply this extension to establish asymptotically optimal estimators in the multiplicative intensity model and in general LAN models. The consideration of estimators in general topological spaces is of importance, since it allows to obtain as immediate consequence the asymptotic efficiency of differentiable functionals $\phi \circ T_n$ of efficient estimators T_n. In order that this idea is fruitful one should consider efficient estimators T_n with values in a function space (like $D[0, 1]$). (We apply this idea to the case where T_n is either the point process N_n itself or the Nelson-Aalen estimator, both on $D[0, 1]$.)

In the second place we establish essentially two different methods of proof for the asymptotic efficiency of estimators in point processes. As examples we use the point process N_n itself and the Nelson-Aalen estimator. One method of proof is the direct way to asymptotic efficiency. We have to establish in the first step the (asymptotic) differentiability of the functional κ which we wish to estimate. Then in a second step we prove that the model is LAN and in a third step that the estimator has a stochastic expansion based on the canonical gradient. This approach is given in Theorem 8 in the iid case and in Theorem 13 in the general LAN case. The second method is to represent the functional which we want to estimate as Hadamard differentiable functional of more simple functionals, where estimators are easy to construct (cf. Proposition 4 and Theorem 5 in the iid case, Theorem 13 in the LAN case). In iid models one obtains particular simple and flexible proofs of asymptotic efficiency of estimators if the underlying model is L^2-differentiable (cf. Theorem 5). We establish L^2-differentiability of the multiplicative point process model in the sense that differentiability of the intensities plus some regularity conditions imply L^2-differentiability of the probability measures. This implies, as is well known, the LAN condition; it also implies the construction of locally most powerful tests and Cramer Rao bounds. "Local" differentiability of point process models (and more generally of semimartingale models) has been established in a recent paper by Jacod (1990) under the sole condition of differentiability of the intensities (also in the case of not multiplicative models). The potentially interesting statistical applications of these results still have to be worked out.

In Section 2 we discuss a sufficient condition for L^2-differentiability in terms of loglikelihood functions. This is useful since for a large class of stochastic processes the density process is an exponential martingale. In Section 3 we consider the estimation in iid models. The reason for treating iid models separately is two-fold. On one hand side one can use the L^2-differentiability property of the underlying model and on the other hand in the case of iid models the relation between efficiency and differentiability is well established in the literature. We establish the L^2-differentiability of the multiplicative point process model in Section 4. In general non-multiplicative point process models we determine an efficient estimator for the integrated expected intensity process. The Nelson–Aalen estimator can be respresented as a Hadamard differentiable functional of this efficient estimator and a further efficient estimator for the expected basis intensity process and, therefore, itself is efficient. In Section 5 we discuss the estimation of asymptotically differentiable functionals with values in topological vectorspaces in the LAN case. This extends the discussion of the relation between efficiency and differentiability given in the iid case by van der Vaart (1989). We consider applications to the estimation of functionals of the intensity process in point process models and in particular give a simple proof of the LAN condition for point process models with differentiable intensities.

2 A Sufficient Condition for L^2-Differentiability

Let $\mathcal{P} \subset M^1(\Omega, \mathcal{A})$ be a family of probability measures on $(\Omega, \mathcal{A})$ dominated by a σ-finite measure μ. A cone $T(P)$ in $L^2(P)$ is called tangent cone in $P \in \mathcal{P}$ if each element $g \in T(P)$ is tangent vector of a L^2-differentiable path in P, i.e. there exists a path $(P_t)_{0 \le t \le 1} \subset \mathcal{P}$ with $P_o = P$ and

$$\int [\frac{1}{t}((\frac{dP_t}{d\mu})^{1/2} - (\frac{dP}{d\mu})^{1/2}) - \frac{1}{2} g (\frac{dP}{d\mu})^{1/2}]^2 d\mu \underset{t \downarrow 0}{\longrightarrow} 0. \tag{2.1}$$

We assume for simplicity reasons that all distributions are pairwise equivalent and denote by $p_t = \frac{dP_t}{d\mu}$, $p = \frac{dP}{d\mu}$ densities w.r.t. μ. Then (2.1) is equivalent to

$$\int \frac{1}{t^2}[(\frac{p_t}{p})^{1/2} - 1 - \frac{1}{2} t g]^2 dP \underset{t \downarrow 0}{\longrightarrow} 0. \tag{2.2}$$

It is well known that (2.2) is equivalent to (cf. Witting (1985), p. 187)

$$\int \frac{1}{t^2} r_t^2 1_{[0,1]}(|r_t|) dP \underset{t \downarrow 0}{\longrightarrow} 0 \tag{2.3}$$

and

$$\int \frac{1}{t^2} |r_t| 1_{(1,\infty)}(|r_t|) dP \underset{t \downarrow 0}{\longrightarrow} 0, \tag{2.4}$$

where $r_t := \frac{p_t}{p} - 1 - tg$. Also the tangent cone $T(P)$ is known to be a subset of $L^2_*(P) = \{h \in L^2(P); \int h \, dP = 0\}$.

The aim of this section is to derive the following criterion for L^2-differentiability in terms of the log likelihood function.

Theorem 1 *Let $(P_t)_{0\le t\le 1} \subset \mathcal{P}$ be a path of pairwise equivalent distributions with log likelihood $\ell_t = \log \frac{p_t}{p}$ and assume that for some $g \in L^2(P)$*

$$\int (\frac{1}{t}\ell_t - g)^2 dP \underset{t\downarrow 0}{\longrightarrow} 0 \tag{2.5}$$

and

$$\limsup_{t\downarrow 0} \int \frac{1}{t^2}\ell_t^2 dP_t \le \int g^2 dP. \tag{2.6}$$

Then $(P_t)_{0\le t\le 1}$ is L^2-differentiable in P with tangent vector g.

Proof: We establish conditions (2.3) and (2.4). Since by assumption (2.5) $r_t = o_p(1)$,

$$\begin{aligned} & \int \frac{1}{t^2}(e^{\ell_t} - 1 - \ell_t)^2\, 1_{[0,1]}(|r_t|) 1_{[0,1]}(|\ell_t - tg|)dP \\ \le\ & \int \frac{1}{t^2}(e^{\ell_t} - 1 - \ell_t)^2\, 1_{[0,3]}(|\ell_t|)dP \\ \le\ & \int \frac{1}{t^2}[\frac{\ell_t^2}{2}(1 + e^{\ell_t})]^2\, 1_{[0,3]}(|\ell_t|)dP \\ \le\ & C\int \frac{1}{t^2}\ell_t^4\, 1_{[0,3]}(|\ell_t|)dP. \end{aligned}$$

For any $0 < \varepsilon < 3$

$$\int \frac{1}{t^2}\ell_t^4\, 1_{(\varepsilon,3]}(|\ell_t|)dP \le 9\int \frac{1}{t^2}\ell_t^2\, 1_{(\varepsilon,3]}(|\ell_t|)dP \underset{t\downarrow 0}{\longrightarrow} 0,$$

since the integral is uniformly integrable by (2.5) and $\ell_t = o_p(1)$. By Lemma 19.1.1 of Pfanzagl and Wefelmeyer (1982) there exists a nullfunction $v : (0,1] \to [0,\infty)$ with $\int \frac{1}{t^2}\ell_t^2\, 1_{(v(t),3]}(|\ell_t|)dP \underset{t\downarrow o}{\longrightarrow} 0$ and, therefore, $\int \frac{1}{t^2}\ell_t^4\, 1_{(v(t),3]}(|\ell_t|)dP \underset{t\downarrow 0}{\longrightarrow} 0$. Since it is easy to see that $\int \frac{1}{t^2}\ell_t^4\, 1_{[0,v(t))}(|\ell_t|)dP \le v(t)^2 \int \frac{1}{t^2}\ell_t^2 dP \underset{t\downarrow 0}{\longrightarrow} 0$ we obtain $\int \frac{1}{t^2}\ell_t^4\, 1_{[0,3]}(|\ell_t|)dP \underset{t\downarrow 0}{\longrightarrow} 0$ and, therefore, $\int \frac{1}{t^2}r_t^2\, 1_{[0,1]}(|r_t|)\, 1_{[0,1]}(|\ell_t - tg|)dP \underset{t\downarrow 0}{\longrightarrow} 0$. Since $\int \frac{1}{t^2}r_t^2\, 1_{[0,1]}(|r_t|)\, 1_{(1,\infty)}(|\ell_t - tg|)dP \le \int (\frac{1}{t}\ell_t - g)^2 dP$, (2.3) is implied by condition (2.5).

In order to establish (2.4) we obtain as in the first part a nullfunction $v : (0,1] \to [0,\infty)$ with

$$\int \frac{1}{t^2}|\ell_t - tg| 1_{(1,\infty)}(|r_t|) 1_{(v(t),\infty)}(|\ell_t - tg|)dP \underset{t\downarrow 0}{\longrightarrow} 0.$$

Since for sufficiently small $t > 0$

$$\begin{aligned} & \int \frac{1}{t^2}|\ell_t - tg| 1_{(1,\infty)}(|r_t|)\, 1_{[0,v(t)]}(|\ell_t - tg|)dP \\ \le\ & \int \frac{1}{t^2}|\ell_t - tg|\, 1_{(1-v(t),\infty)}(|e^{\ell_t} - 1 - \ell_t|)\, 1_{[0,v(t)]}(|\ell_t - tg|)dP \\ \le\ & \frac{v(t)}{t^2}\int 1_{(1-v(t),\infty)}(|e^{\ell_t} - 1 - \ell_t|)dP \end{aligned}$$

$$\leq \frac{v(t)}{t^2} \int 1_{(1/2,\infty)}(|\ell_t|) dP$$
$$\leq 4v(t) \int \frac{1}{t^2} \ell_t^2 \, dP \underset{t\downarrow 0}{\longrightarrow} 0,$$

we obtain $\int \frac{1}{t^2} |\ell_t - tg| 1_{(1,\infty)}(|r_t|) dP \underset{t\downarrow 0}{\longrightarrow} 0$. Furthermore,

$$\begin{aligned} & \int \frac{1}{t^2} |e^{\ell_t} - 1 - \ell_t| \, 1_{(1,\infty)}(|r_t|) dP \\ \leq & \int \frac{1}{t^2} \frac{\ell_t^2}{2} (1 + e^{\ell_t}) 1_{(1,\infty)}(|r_t|) dP \\ = & \frac{1}{2} \int \frac{1}{t^2} \ell_t^2 \, 1_{(1,\infty)}(|r_t|) dP \\ + & \frac{1}{2} \int \frac{1}{t^2} \ell_t^2 \frac{p_t}{p} \, 1_{(1,\infty)}(|r_t|) dP. \end{aligned}$$

By (2.5) the integrand in the first term is uniformly integrable and converges stochastically to zero; so the first term tends to zero. We next prove the uniform integrability of the integrand of the second term. By Vitali's theorem and (2.6) it is sufficient to show that $\frac{1}{t^2} \ell_t^2 \frac{p_t}{p} - g^2 = o_p(1)$. We decompose $\frac{1}{t^2} \ell_t^2 \frac{p_t}{p} - g^2 = \frac{1}{t^2} \ell_t^2 r_t + \frac{1}{t^2} \ell_t^2 - g^2 + \frac{1}{t^2} \ell_t^2 tg$. For $\varepsilon > 0$ holds:

$$P(\frac{1}{t^2} \ell_t^2 \geq M) \leq \frac{1}{M} \int \frac{\ell_t^2}{t^2} dP \underset{M\to\infty}{\longrightarrow} 0.$$

Since by (2.5) this holds uniformly in t, $P(\frac{1}{t^2} \ell_t^2 \geq M(\varepsilon)) \leq \frac{\varepsilon}{2}$ for all $t > 0$ and some $M(\varepsilon) > 0$. Since $r_t = o_p(1)$ this implies for small t and any $\delta > 0$: $P(|\frac{1}{t^2} \ell_t^2 r_t| > \delta) \leq P(\frac{1}{t^2} \ell_t^2 \geq M(\varepsilon)) + P(\frac{1}{t^2} \ell_t^2 < M(\varepsilon),\ \frac{1}{t^2} \ell_t^2 |r_t| > \delta) \leq \frac{\varepsilon}{2} + P(M(\varepsilon)|r_t| > \delta) \leq \varepsilon$. Similarly, also the third term converges to zero stochastically. $\square$

We shall need the following lemma to apply (2.5) in examples.

Lemma 2.1 *In the situation of Theorem 1 let $\int \ell_t \, dP_t \underset{t\downarrow 0}{\longrightarrow} 0$, then $H(P_t, P) \underset{t\downarrow 0}{\longrightarrow} 0$, H the Hellinger distance. If, furthermore, $f = (f(t))_{0 \leq t \leq 1}$ is a stochastic process with $\|E_P f^2\|_\infty < \infty$ and $\limsup_{t\downarrow 0} \|E_{P_t} f^2\|_\infty < \infty$, then $\|E_{P_t} f - E_P f\|_\infty \underset{t\downarrow 0}{\longrightarrow} 0$.*

Proof: Note that $\int \ell_t \, dP_t = \int p_t \ell n \frac{p_t}{p} d\mu = I(P_t, P)$ is the Kullback-Leibler I-divergence. By Csiszar's (1966) inequality holds $\|P_t - P\| \leq 2(I(P_t, P))^{1/2}$. On the other hand by Lemma 2.15 in Strasser (1985) holds $H^2(P_t, P) \leq 2\|P_t - P\|$. Therefore, $H(P_t, P) \leq 2(I(P_t, P))^{1/4}$. For the second part we use the inequalities

$$\begin{aligned} & |\int f(s) dP_t - \int f(s) dP| \\ = & |\int f(s)((\frac{p_t}{p})^{1/2} + 1)((\frac{p_t}{p})^{1/2} - 1) dP| \end{aligned}$$

$$\leq \left(\int f^2(s)((\frac{p_t}{p})^{1/2}+1)^2 dP \int((\frac{p_t}{p})^{1/2}-1)^2 dP\right)^{1/2}$$
$$\leq 2\{(\|E_{P_t} f^2\|_\infty + \|E_P f^2\|_\infty)\}^{1/2} H(P_t, P) \underset{t\downarrow 0}{\longrightarrow} 0.$$

□

3 Efficient Estimation in i.i.d. Point Process Models

In i.i.d. models efficient estimation of differentiable functionals $\kappa : \mathcal{P} \to B$ with values in general topological vector spaces B has been investitaged by several authors; we refer in particular to Millar (1990), Strasser (1985), van der Vaart (1988) and Le Cam (1987). For theorems of the type: efficiency of estimators (T_n) implies that of $(\phi \circ T_n)$ for Hadamard differentiable functionals ϕ (tangentially to certain supporting sets) cf. van der Vaart (1988) and Gill (1986).

Let (B, d) be a metric topological vector space with σ-algebra $\mathcal{B}$ satisfying the standard conditions:

1. Translation and scalar multiplication are measurable;
2. $\mathcal{B}$ contains the balls and is contained in the Borel σ-algebra;
3. each separable probability measure L on $(B, \mathcal{B})$ is uniquely determined by the marginals $(b^*(L))$, $b^* \in B^* = B^*_\tau$ the dual w.r.t. topology τ generated by d, b^* $\mathcal{B}$-measurable.

These conditions are satisfied for $B = D[0,1]$ supplied with supremum metric and $\mathcal{B} = \sigma(\pi_t,\ 0 \leq t \leq 1)$ the σ-algebra, generated by the projections. Let $T(P) = T(P, \mathcal{P})$ be a tangent cone in $L^2(P)$ and let $\kappa : \mathcal{P} \to B$ be a differentiable functional i.e. $\forall g \in T(P)$ there is a path (P_t) in P with tangent vector g and $\frac{1}{t}(\kappa(P_t) - \kappa(P)) \to \kappa'_P(g)$, where $\kappa'_P : \operatorname{lin} T(P) \to B$ is continuous linear and so can be extended to the closure of lin $T(P)$ in $L^2(P)$. $\dot{\kappa}_{b^*}(\cdot, P) \in L^2(P)$ is called gradient in direction $b^* \in B^*_\tau$ if

$$b^* \circ \kappa'_p(g) = \int g \dot{\kappa}_{b^*}(\cdot, P) dP, \forall g \in T(P). \tag{3.1}$$

The projection $\tilde{\kappa}_{b^*}(\cdot, P)$ of a gradient $\dot{\kappa}_{b^*}(\cdot, P)$ on $\overline{\operatorname{lin}} T(\mathcal{P})$ is called canonical gradient.

We assume the existence of a separable probability measure N on $(B, \mathcal{B})$ with $b^*(N) = N(0, \|\tilde{\kappa}_{b^*}(\cdot, P)\|_P^2)$ for all $b^* \in B^*_\tau$, then the convolution theorem and minimax theorem hold in the asymptotic iid model $(\mathcal{P}^n)$ (cf. van der Vaart (1988) or in a different formulation Strasser (1985) and Millar (1988) and a regular estimator sequence is efficient if it has (normalized) N as its asymptotic distribution. This is equivalent to the condition that $\sqrt{n}(T_n - \kappa(P))$ is tight and $b^* \circ T_n$ is efficient for

$b^* \circ \kappa,\ b^* \in B_\tau^*,\ \mathcal{B}$-measurble (we assume the usual rate $\sqrt{n}$ at this place). The last condition is known to be equivalent to a stochastic expansion:

$$\sqrt{n}(b^* \circ T_n - b^* \circ \kappa(P)) = n^{-1/2} \sum_{j=1}^{n} \tilde{\kappa}_{b^*}(x_j, P) + o_p(1). \tag{3.2}$$

In (3.2) one can restrict to "generating" subsets $B' \subset B_\tau^*$; in the case $B = D[0,1]$ one can restrict to $b^* = \pi_t,\ t \in [0,1]$ (cf. Theorem 4.9 of van der Vaart (1988)).

Lemma 3.1 *Let $T(P) \subset L^2(P)$ be a tangent cone with $\overline{lin}T(P) = L_*^2(P)$ and let $f = (f_t)_{0 \le t \le 1}$ be a stochastic process in $D[0,1]$ such that*

$$(f_t) \text{ is uniformly integrable} \tag{3.3}$$

and

$$\sup\{\int f_s^2\, dQ; Q \in \mathcal{P}, H(Q,P) < \varepsilon,\ s \in [0,1]\} < \infty \tag{3.4}$$

for some $\varepsilon > 0$, or

$$\sup\{\int f_s^2\, dQ; Q \in \mathcal{P}, V(Q,P) < \varepsilon,\ s \in [0,1]\} < \infty$$

for some $\varepsilon > 0$, V the sup-metric. If $(f^{(j)})$ is a sequence of i.i.d. copies of f, then $T_n = \frac{1}{n}\sum_{j=1}^n f^{(j)}$ is asymptotically efficient for $\kappa : \mathcal{P} \to D[0,1]$, $\kappa(P)_s = \int f_s\, dP$, $s \in [0,1]$ if $(\sqrt{n}(T_n - \kappa(P)))$ is tight w.r.t. P.

Proof: The proof of Lemma (5.21) in van der Vaart (1988) and conditions (3.3), (3.4) imply that κ is differentiable with canonical gradient $\tilde{\kappa}_{\pi_t}(f,P) = f_t - \int f_t\, dP$. Since

$$\begin{aligned} & \sqrt{n}(\pi_t \circ T_n - \pi_t \circ \kappa(P)) \\ = & \frac{1}{\sqrt{n}} \sum_{j=1}^{n} (f_t^{(j)} - \int f_t^{(j)} dP) \\ = & \frac{1}{\sqrt{n}} \sum_{j=1}^{n} \tilde{\kappa}_{\pi_t}(f^{(j)}, P), \end{aligned}$$

(T_n) is asymptotically efficient. □

Since convergence in distribution w.r.t. the sup-norm is equivalent to convergence w.r.t. the Skorohod topology if the limiting process is continuous (cf. Pollard (1984), p. 137), it is sufficient that

$$\sqrt{n}(T_n - \kappa(P)) \xrightarrow{\mathcal{D}} V, \tag{3.5}$$

V a process with continuous path's.

For the application of Lemma 3 to the i.i.d. point process model $(N, \mathcal{F})$ with intensity λ_α w.r.t. $P_\alpha,\ \alpha \in I$, we formulate the following conditions:

C.1 $\overline{\mathrm{lin}\, T(P_\alpha)} = L^2_*(P_\alpha)$;

C.2 $\sup\{\int_0^1 E_\beta \lambda^i_\beta(s)ds;\ \beta \in I,\ H(P_\beta, P_\alpha) < \varepsilon\} < \infty$ for some $\varepsilon > 0,\ i = 1, 2$;

C.3 λ_α has caglad path's and $\sqrt{n}(\frac{1}{n}\lambda_{n,\alpha} - E_\alpha\lambda_\alpha) \xrightarrow{\mathcal{D}} Y$, a process with continuuos path's (convergence in $G[0,1]$, the class of caglad processes with Skorohod topology).

Condition 1 will be established in Section 4 in the case of multiplicative intensities. The CLT in C.3 is a consequence of the conditions: $\exists \alpha' > \frac{1}{2},\ \beta > 1$, such that for $0 \le s \le t \le u \le 1$:

$$\begin{aligned} E(\lambda_\alpha(u) - \lambda_\alpha(t))^2 &\le (G(u) - G(t))^{\alpha'} \\ E(\lambda_\alpha(u) - \lambda_\alpha(t))^2(\lambda_\alpha(t) - \lambda_\alpha(s)^2 &\le (F(u) - F(s))^\beta \end{aligned} \tag{3.6}$$

for some continuous, monotonically nondecreasing functions F, G on $[0,1]$ (cf. Hahn (1978)). Consider next the estimation of the functional

$$\kappa_1(P_\alpha) = E_\alpha N = \int_0^{\cdot} E_\alpha \lambda_\alpha(s)ds.$$

Proposition 2 *Under conditions C.1, C.2, C.3 $\kappa_1 : \mathcal{P} \to (D[0,1], \|\ \|_\infty)$ is differentiable in P_α and $T_{1,n} = \frac{1}{n}N_n = \frac{1}{n}\sum_{i=1}^n N^{(i)}$ is asymptotically efficient for κ_1.*

Proof: By Lemma 3, we have to bound $\int N_s^2 dP_\beta$ uniformly in a neighbourhood of P_α. With $A_\beta(t) = \int_0^t \lambda_\beta(s)ds$ and $M_\beta(t) = N_t - A_\beta(t)$ we obtain for $s \in [0,1]$

$$\begin{aligned} \int N_s^2 dP_\beta &\le E_\beta N_1^2 = \int (M_\beta(1) + A_\beta(1))^2 dP_\beta \\ &\le 2(\int (M_\beta(1))^2 dP_\beta + \int (A_\beta(1))^2 dP_\beta) \\ &= 2(\int\int_0^1 \lambda_\beta(s)ds\, dP_\beta + \int (\int_0^1 \lambda_\beta(s)ds)^2 dP_\beta \\ &\le 2[\int_0^1 E_\beta\lambda_\beta(s)ds + \int\int_0^1 \lambda^2_\beta(s)ds\, dP_\beta] \\ &< \infty \text{ by C.2 if } H(P_\beta, P_\alpha) < \varepsilon. \end{aligned} \tag{3.7}$$

In the next step we have to establish tightness of $(\sqrt{n}(T_{1,n} - \kappa_1(P_\alpha)))$ w.r.t. P_α.

$$\sqrt{n}(T_{1,n} - \kappa_1(P_\alpha))_t = \frac{1}{\sqrt{n}}\sum_{i=1}^n M_\alpha^{(i)}(t) + \int_0^t \sqrt{n}(\frac{1}{n}\lambda^{(s)}_{n,\alpha} - E_\alpha\lambda_{n,\alpha}(s))ds, \tag{3.8}$$

where $\lambda_{n,\alpha}(s) = \sum_{i=1}^n \lambda_\alpha^{(i)}(s)$ is the intensity of N_n. For the convergence of the first term we apply the CLT of Rebolledo (cf. [10], [11], [21]). For the predictable variation

$$\begin{aligned} &< \frac{1}{\sqrt{n}}\sum_{i=1}^n M_\alpha^{(i)}(t) >_t = \frac{1}{n}\sum_{i=1}^n < M_\alpha^{(i)} >_t \\ = &\ \frac{1}{n}\sum_{i=1}^n \int_0^t \lambda_\alpha^{(i)}(s)ds = \int_0^t \frac{1}{n}\sum_{i=1}^n \lambda_\alpha^{(i)}(s)ds \\ = &\ \int_0^t \lambda_{n,\alpha}(s)ds. \end{aligned} \tag{3.9}$$

By the strong law of large numbers in $L^1[0,1]$ a.s.: $\lambda_{n,\alpha} \to \int_0 E_\alpha \lambda_\alpha(s)ds = \kappa_1(P_\alpha)$ in $L^1[0,1]$, which implies that

$$\int_0^t \lambda_{n,\alpha}(s)ds \to \int_0^t E_\alpha \lambda_\alpha(s)ds =: \Lambda(t).$$

Obviously, $\Lambda \in C[0,1]$.

For the Lindeberg condition we consider

$$|\Delta \frac{1}{\sqrt{n}} \sum_{i=1}^n M_\alpha^{(i)}(t)| = \frac{1}{\sqrt{n}} \Delta N_{n,t}$$

and, therefore,

$$\begin{aligned} & E_\alpha[\sum_{0\le t\le 1} (\Delta \frac{1}{\sqrt{n}} \sum_{i=1}^n M_\alpha^{(i)}(t))^2 \, 1_{\{|\Delta \frac{1}{\sqrt{n}} \sum_{i=1}^n M_\alpha^{(i)}(t)|>\varepsilon\}}] \qquad (3.10) \\ = \;& E_\alpha[\sum_{0\le t\le 1} (\frac{1}{\sqrt{n}} \Delta N_{N,t})^2 \, 1_{\{\frac{1}{\sqrt{n}}\Delta N_{n,t}>\varepsilon\}}] \\ \le \;& E_\alpha(\sum_{0\le t\le 1} \frac{1}{n} \Delta N_{n,t} \, 1_{\{\frac{1}{\sqrt{n}}>\varepsilon\}}) \quad (\text{since } \; \Delta N_{n,t} \in \{0,1\}) \\ = \;& 1_{\{\frac{1}{\sqrt{n}}>\varepsilon\}} \frac{1}{n} E_\alpha N_{n,1} \\ = \;& 1_{\{\frac{1}{\sqrt{n}}>\varepsilon\}} \int_0^t E_\alpha \lambda_\alpha(s)ds \to 0. \end{aligned}$$

Therefore, the CLT of Rebolledo implies weak convergence of the first term in (3.8).

For the second term of (3.8) by assumption C.3 the integrand converges to a process with continuous path's on $[0,1]$. By the a.s. representation theorem of Skorohod there exists a version $\tilde{Y}_n$ of $Y_n := \sqrt{n}(\frac{1}{n}\lambda_{n,\alpha}(\cdot) - E\lambda_\alpha(\cdot))$ and $\tilde{Y}$ of Y such that $\|\tilde{Y}_n - \tilde{Y}\|_\infty \to 0$ a.s. This implies that $\int_0 \tilde{Y}_n(s)ds \to \int_0 \tilde{Y}(s)ds$ a.s. and, therefore, weak convergence of the second term.

From the decomposition CLT of Hahn (1978) we therefore can conclude that the sum of both terms $(\sqrt{n}(T_{1,n} - \kappa_1(P_\alpha)))$ converges to a process with continuous path's (which is easy to identify by the finite dimensional distributions). □

Remark:

a) In the multiplicative intensity model $\lambda_\alpha = \alpha\lambda$ the conditions can be modified. If $|\lambda| \le M < \infty$, then C.2 can be replaced by

   C.2′ $\sup\{\int_0^1 \beta(s)ds; \beta \in I, H(P_\beta, P_\alpha) < \varepsilon\} < \infty$.

b) In the case $\lambda_\alpha = \alpha\lambda$ consider the estimation of $\kappa_2(P_\alpha) = E_\alpha\lambda$. Under condition C.1,

   C.2″ $\sup\{E_\beta \lambda_s^2; s \in [0,1], H(P_\beta, P_\alpha) < \varepsilon\} < \infty$ and

   C.3″ $\sqrt{n}(\frac{1}{n}\lambda_n - E_\alpha\lambda) \xrightarrow{\mathcal{D}} Y$, where $\lambda_n(s)$, a process with continuous path's,

$$T_n = \frac{1}{n}\lambda_n \quad \text{is asymptotically efficient for} \quad \kappa_2. \tag{3.11}$$

The proof is similar to that of Proposition 4.

c) Extensions of Proposition 4 to the a.s. efficiency of

$$\frac{1}{n}\dot{\int_0} h(s)dN_{n,s} \quad \text{for} \quad \kappa(P_\alpha) = \dot{\int_0} h(s)E_\alpha\lambda_\alpha(s)ds$$

for a known function h are obvious.

$\square$

We next give an application to the efficient estimation of the compensator in the multiplicative intensity model

Theorem 3 *Under assumptions C.1, C.2, C.2″, C.3″, and*

C.4 $\frac{1}{E_\alpha\lambda}$ *is of bounded variation*

in the multiplicative intensity model, the functional $\kappa : \mathcal{P} \to (D[0,1], \| \ \|_\infty)$, $\kappa(P_\alpha) = \dot{\int_0}\alpha(s)ds$ *is differentiable in* P_α *and the Nelson-Aalen estimator* $T_n = \dot{\int_0}\frac{1}{\lambda_n(s)}1_{\{\lambda_n(s)>0\}}dN_{n,s}$ *is asymptotically efficient for* κ *in* P_α.

Proof: With $E_\alpha N_t = \int_0^t \alpha(s)E_\alpha\lambda_s ds$, we obtain

$$\begin{aligned}
\kappa(P_\alpha) &= \dot{\int_0}\frac{1}{E_\alpha\lambda_s}\alpha(s)E_\alpha\lambda_s ds \\
&= \dot{\int_0}\frac{1}{E_\alpha\lambda_s}d(E_\alpha N_s) = \phi(\frac{1}{E_\alpha\lambda}, E_\alpha N),
\end{aligned}$$

where $\phi : G[0,1] \times D[0,1] \to D[0,1]$ is defined by $\phi(x,y) = \dot{\int_0}x\, dy^\uparrow$, $y^\uparrow$ the least monotonically nondecreasing majorant of y in $D[0,1]$ and where $G[0,1]$ are the caglad functions, $D[0,1]$ (as usual) the cadlag functions. The proof follows from the following steps:

1. For $y \in C[0,1]$ nondecreasing ϕ is Hadamard-differentiable in (x,y) tangentially to $G[0,1] \times K_y[0,1]$, where $K_y[0,1] = \{k \in C[0,1]; k$ is constant on intervals where y is constant$\}$ and $\phi'_{(x,y)}(h,k) = \dot{\int_0}x\, dk + \dot{\int_0}h\, dy$, $h \in G[0,1], k \in K_y[0,1]$ ($\dot{\int_0}x\, dk$ defined by partial integration).

   1 is an extension of Lemma 3 of Gill (1976), where some additional assumptions are made. We omit the somewhat technical involved proof (cf. Holtrode (1990)), which consists in the proof of the following four steps:

   (a) $\uparrow$ is Hadamard-differentiable in y tangentially to $K_y[0,1]$.

(b) $h_n, h \in G[0,1]$, $h_n \to h$, $y_n, y \in D[0,1]$, $y_n \to y$ and $\overline{\lim} \int_0^1 d(\operatorname{Var} y_n) < \infty$ implies $\int_0^{\cdot} h_n dy_n \to \int_0^{\cdot} h\, dy$ in $(D[0,1], \|\ \ \|_\infty)$ (cf. Lemma 2 of Gill (1986)).

(c) The conditions of (b) hold for $y_n = (y + t_n k_n)^\uparrow$.

(d) If $h_n \to h$ in $G[0,1]$ and $k_n \to k$ in $D[0,1]$, $t_n \to 0$, then

$$\begin{aligned} & \|\frac{1}{t_n}[\phi(x + t_n h_n, y_n + t_n k_n) - \phi(x, y) \\ - & \int_0^{\cdot} x\, d(t_n k_n) - \int_0^{\cdot} t_n h_n\, dy]\|_\infty \\ \leq & \|\frac{1}{t_n}[(y + t_n k_n)^\uparrow - y^\uparrow] - k_n\|_\infty \ \|x\|_\infty \\ + & \|\int_0^{\cdot} \frac{1}{t_n}[(y + t_n k_n)^\uparrow - y^\uparrow - t_n k_n] dx\ \|_\infty \\ + & \|\int_0^{\cdot} h_n\, d(y + t_n k_n)^\uparrow - \int_0^{\cdot} h_n dy\|_\infty \to 0, \end{aligned}$$

the first inequality follows from some calculations.

2. $T_{1,n} = \frac{1}{n} N_n$ is (by Proposition 4) asymptotically efficient in P_α for $\kappa_1(P_\alpha) = E_\alpha N$.

3. $T_{2,n} = \frac{n}{\lambda_n} 1_{\{\lambda_n > 0\}}$ is asymptotically efficient in P_α for $\kappa_3(P_\alpha) = \frac{1}{E_\alpha \lambda}$.

   Proof. By Remark b, (3.11) $(\frac{1}{n}\lambda_n)$ is as. efficient for $\kappa_2(P_\alpha) = E_\alpha \lambda$ in P_α. Since $\|\sqrt{n}\frac{1}{n} 1_{\{\lambda_n = 0\}}\|_\infty \leq \frac{1}{\sqrt{n}} \to 0$ also $\tilde{T}_n = \frac{1}{n}\lambda_n + \frac{1}{n} 1_{\{\lambda_n = 0\}}$ is as. efficient. $f(x) = \frac{1}{x}$ is Hadamard-differentiable in x on $\{x \in G[0,1]; x > 0\}$ with $f'_x(h) = -\frac{h}{x^2}$ if $\|\frac{1}{x}\|_\infty \leq \delta$. Therefore, by C.4 κ_3 is Hadamard-differentiable in P_α with derivative $(\kappa_3)'_{P_\alpha}(g) = -\frac{E_\alpha(\lambda g)}{(E_\alpha \lambda)^2}$ and $f(\tilde{T}_n)$ is as. efficient for κ_2 in P_α. Since by C.3" $\|\sqrt{n} n\, 1_{\{\lambda_n = 0\}}\|_\infty \xrightarrow{P_\alpha} 0$, this implies a.s. efficiency of $T_{2,n}$.

   Since $y = E_\alpha N$ is monotonically nondecreasing and $\frac{1}{E_\alpha \lambda}$ is of bounded variation, we obtain by 1 that ϕ is differentiable in $(\frac{1}{E_\alpha \lambda}, E_\alpha N)$ tangentially to $G[0,1] \times K_y[0,1]$, $y := E_\alpha N$. So we next have to show that:

4. The limit Z of $\sqrt{n}(\frac{1}{n} N_n - E_\alpha N)$ is concentrated on $K_y[0,1]$.

   Proof. For $s \leq t$: $\sqrt{n}(\frac{1}{n} N_{n,t} - E_\alpha N_t) - \sqrt{n}(\frac{1}{n} N_{n,s} - E_\alpha N_s) \xrightarrow{\mathcal{D}} Z_t - Z_s$. If $y = E_\alpha N$ is constant on $[s, u)$, then for $t \in [s, u)$, $N_t \geq N_s$ and $E_\alpha N_t = E_\alpha N_s$, i.e. $N_t = N_s$ a.s. and so, $N_{n,t} = N_{n,s}[P_\alpha]$ which implies $Z_t = Z_s[P_\alpha]$. Since there are at most countable many constancy intervals, this implies that $Z \in K_y[0,1]$ a.s.

5. $E_\alpha g N \in K_y[0,1], \forall g \in T(P_\alpha)$.

Proof. If for $g \in T(P_\alpha), E_\alpha N_t = E_\alpha N_s,\ s \le t$, then $1_{[s,t]}(u)\alpha(u) = 0[\lambda\!\!\lambda^1]$ and

$$\begin{aligned} E_\alpha g N_t &= E_\alpha \int_0^t g\alpha(u)\lambda_u du \\ &= \int_0^t \alpha(u) E_\alpha(g\lambda_u) du = \int_0^s \alpha(u) E_\alpha(g\lambda_u) du \\ &= E_\alpha g N_s. \end{aligned}$$

Therefore, $E_\alpha g N \in K_y[0,1]$ a.s.

By Theorem 4.11 of van der Vaart (1988), therefore, $T_n = \phi(T_{1,n}, T_{2n})$ is a.s. efficient for $\kappa = \phi(\kappa_2, \kappa_1)$. □

Again we also can use the modified conditions as in the remark after Proposition 4.

In the following example we establish the conditions of Theorem 5; for some different discussion of this example we refer to [24], [18], [5].

Example 3.1 (Censoring Model)
Let X be a real rv'e with df F, hazard rate $h = \frac{f}{1-F}$ caglad on $\{F < 1\}$. Let C be a positive censoring r.v. with continuous df G independent of X and $Z = \min\{X, C\}$, $F(1) < 1$, $G(1) < 1$ and $\delta = 1_{\{X \le C\}}$ known. Let $N_t = 1_{\{Z \le t, \delta = 1\}}$, then N has P_h-intensity $\lambda_h(t) = h(t) 1_{\{Z \ge t\}} =: h(t)\lambda_t$ and $M_h(t) = N_t - \int_0^t h(s) 1_{\{Z \ge s\}} ds$ is a martingale. The Nelson-Aalen estimator estimates the cumulative hazard function $\Lambda = \int_0^{\cdot} h(s) ds$.

We next establish the conditions of Theorem 5.

C.1 $\overline{\operatorname{lin} T(P_h)} = L^2_*(P_h)$ for $P_h \in \mathcal{P}$.
Proof. For $g \in \mathcal{L}^2_*(P_h),\ c = \frac{1}{4}\int g^2 dP_h \le 1$ define $dP_t = (\frac{1}{2}tg + \sqrt{1 - t^2 c})^2 dP_h$, which is a L^2-differentiable path in $\mathcal{P}$ with tangent vector g if g is caglad. So $\{g \in \mathcal{L}^2_*(P_h);\ g \text{ caglad}\} \subset T(P_h)$. By Lusin's theorem the caglad elements are dense in $\mathcal{L}^2(P_h)$, which implies 1 (cf. also Wellner (1982)).

C.2$'$ For any hazard rate h,

$$\begin{aligned} \int_0^1 h(s) ds &= \int_0^1 \frac{g(s)}{1 - G(s)} \le \int_0^1 \frac{g(s)}{1 - G(1)} ds \\ &\le \frac{1}{1 - G(1)} < \frac{1}{1 - F(1)} + \delta < \infty \text{ if } V(P_F, P_G) < \varepsilon. \end{aligned}$$

C.2$''$ is obvious, since $\lambda_t = 1_{\{Z \ge t\}}$ is bounded.

C.3$''$ Let $X_t := 1_{\{Z \ge t\}} - P_h(Z \ge t)$, then $a(t,u) = E_h(X_u - X_t)^2 = \operatorname{Var}_h 1_{\{t \le Z < u\}} \le P_h(t \le Z < u)$ and for $s < t < u$, $b(s,t,u) = E_h(X_u - X_t)^2 (X_t - X_s)^2 \le$

$2P_h(s \le Z < t)P_h(t \le Z < u)$. So with $A(t) = B(t) = -2P_h(Z \ge t)$, $\alpha = \beta = 1$ holds

$$\begin{aligned} a(t,u) &\le (A(u) - A(t))^\alpha \\ b(s,t,u) &\le (B(u) - B(t))^\beta (B(t) - B(s))^\beta, \end{aligned}$$

which implies the CLT for (λ_n) by Theorem 2 of Hahn (1978).

C.4 If H is the distribution function of Z, then $1 - H = (1-F)(1-G)$ is continuous. Since $0 < F(1), G(1) < 1$, $\frac{1}{E_h \lambda_s} = \frac{1}{1-H(s)}$ is monotonically nondecreasing and bounded.

Alltogether, by Theorem 5 the Nelson-Aalen estimator is a.s. efficient.

4 L^2-Differentiability of the Point Process Model With Multiplicative Intensities

In this section we establish the L^2-differentiability condition for the model with multiplicative intensities, and, therefore, condition C.1 of Section 3 for these models. Let $\alpha \in I$ be a fixed element and let $\alpha_t = \alpha_t(s)$ be elements of I with corresponding probabilities $(P_{\alpha_t})_{0 \le t \le 1} \subset \mathcal{P}$, a path in $\mathcal{P}$ through P_α. $\alpha\lambda\!\!\lambda^1$ denotes the measure with density α w.r.t. Lebesgue measure $\lambda\!\!\lambda^1$.

Theorem 4 *Let $(P_t) = (P_{\alpha_t})$ satisfy the following conditions:*

$$\int_0^1 \alpha(s)\lambda(s)ds \le K < \infty,\ \|E_\alpha \lambda^2\|_\infty < \infty, \tag{4.1}$$

$$\limsup_{t\downarrow 0} \|\frac{\alpha}{\alpha_t}\|_\infty < \infty,\ \limsup_{t\downarrow 0} \|\frac{\alpha_t}{\alpha}\|_\infty < \infty \quad and \quad \limsup_{t\downarrow 0} \|E_t \lambda^2\|_\infty < \infty. \tag{4.2}$$

For some $v \in \mathcal{L}^2(\alpha\lambda\!\!\lambda^1)$ holds

$$\int_0^1 [\frac{1}{t}(\frac{\alpha_t(s)}{\alpha(s)} - 1) - v(s)]^2 \alpha(s) ds \underset{t\downarrow 0}{\longrightarrow} 0. \tag{4.3}$$

For some $p > 1$ holds:

$$\limsup_{t\downarrow 0} \int_0^1 [\frac{1}{t}(\frac{\alpha_t(s)}{\alpha(s)} - 1)]^{2p} \alpha(s) ds < \infty, \tag{4.4}$$

then (P_t) is L^2-differentiable in P_α with tangent vector $\int_0^1 v(s)dM_\alpha(s)$.

Proof: For the proof we establish conditions (2.5), (2.6) of Theorem 1 in several steps.
Step 1. There exist $A_t \in \mathbb{R}^1$, $\limsup_{t\downarrow 0} A_t < \infty$, such that for caglad functions $f_t = f_t(s)$

$$E_t(\int_0^1 f_t dN)^2 \le A_t \int_0^1 f_t^2(s)\alpha(s)ds. \tag{4.5}$$

Proof. We shall omit integration variables if no problems arise. With $M_t = M_{\alpha_t}$,

$$\begin{aligned}
E_t(\int_0^1 f_t dN)^2 &= E_t[\int_0^1 f_t dM_t + \int_0^1 f_t \alpha_t \lambda]^2 \\
&\leq 2E_t(\int_0^1 f_t dM_t)^2 + 2E_t(\int_0^1 f_t \alpha_t \lambda)^2 \\
&\leq 2E_t(\int_0^1 f_t^2 \alpha_t \lambda ds) + 2E_t(\int_0^1 f_t^2 \alpha_t \lambda ds)(\int_0^1 \alpha_t \lambda ds) \\
&\leq 2\|\frac{\alpha_t}{\alpha}\|_\infty(\int_0^1 f_t^2 \alpha ds)\|E_t\lambda\|_\infty \\
&+ 2\|\frac{\alpha_t}{\alpha}\|_\infty^2 \|E_t\lambda\|_\infty(\int_0^1 f_t^2 \alpha ds)(\int_0^1 \alpha ds)
\end{aligned}$$

Similarly,

$$E_\alpha(\int_0^1 f_t dN)^2 \leq B\int_0^1 f_t^2 \alpha ds. \tag{4.6}$$

Step 2.

$$\int(\frac{1}{t}\ell_t - \int_0^1 v\, dM_\alpha)^2 dP_\alpha \underset{t\downarrow 0}{\longrightarrow} 0. \tag{4.7}$$

Proof. Let $R(x) = \log(1+x) - x,\ x > -1$, then

$$|R(x)| \leq x^2 \frac{1}{1+x},\ x > -1. \tag{4.8}$$

By Liptser, Shiryayev (1978), we obtain from (4.2) that P_t and P_α are equivalent for small t with log likelihood function

$$\begin{aligned}
\ell_t &= \int_0^1 \log(\frac{\alpha_t}{\alpha})dN - \int_0^1(\alpha_t - \alpha)\lambda ds \qquad (4.9)\\
&= \int_0^1(\frac{\alpha_t}{\alpha} - 1)dN + \int_0^1 R(\frac{\alpha_t}{\alpha} - 1)dN - \int_0^1(\frac{\alpha_t}{\alpha} - 1)\alpha\lambda ds \\
&= \int_0^1(\frac{\alpha_t}{\alpha} - 1)dM_\alpha + \int_0^1 R(\frac{\alpha_t}{\alpha} - 1)dN.
\end{aligned}$$

We have to consider the convergence to zero of the second term normalized by $\frac{1}{t}$.

$$\begin{aligned}
&E_\alpha[\frac{1}{t}\int_0^1 R(\frac{\alpha_t}{\alpha} - 1)dN]^2 \qquad (4.10)\\
\leq\ & E_\alpha[\int_0^1 \frac{1}{t}(\frac{\alpha_t}{\alpha} - 1)^2 \frac{\alpha}{\alpha_t} dN]^2 \\
\leq\ & B\int_0^1 \frac{1}{t^2}(\frac{\alpha_t}{\alpha} - 1)^4 (\frac{\alpha}{\alpha_t})^2 \alpha ds \quad \text{by step 1} \\
\leq\ & B(\int_0^1 \frac{1}{t^{2p}}(\frac{\alpha_t}{\alpha} - 1)^{2p}\alpha ds)^{1/p}(\int_0^1(\frac{\alpha_t}{\alpha} - 1)^{2q}(\frac{\alpha}{\alpha_t})^{2q}\alpha ds)^{1/q} \\
\leq\ & B(\int_0^1 \frac{1}{t^{2p}}(\frac{\alpha_t}{\alpha} - 1)^{2p}\alpha ds)^{1/p}\{\|\frac{\alpha}{\alpha_t}\|_\infty^{2q}\|\frac{\alpha_t}{\alpha} - 1\|_\infty^{2q-2}\int_0^1(\frac{\alpha_t}{\alpha} - 1)^2\alpha ds\}^{1/q} \underset{t\downarrow 0}{\longrightarrow} 0,
\end{aligned}$$

by assumptions (4.1) - (4.4).

(4.7) is now a consequence of

$$
\begin{aligned}
& E_\alpha(\frac{1}{t}\int_0^1(\frac{\alpha_t}{\alpha}-1)dM_\alpha - \int_0^1 v dM_\alpha)^2 \qquad (4.11)\\
= \;& E_\alpha[\int_0^1(\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)-v)dM_\alpha]^2\\
= \;& \int_0^1(\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)-v)^2\alpha E_\alpha\lambda ds\\
\le \;& \|E_\alpha\lambda\|_\infty\int_0^1[\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)-v)]^2\alpha ds \underset{t\downarrow 0}{\longrightarrow} 0
\end{aligned}
$$

by (4.4).
Step 3. $\int \ell_t^2 dP_t \underset{t\downarrow 0}{\longrightarrow} 0.$
Proof.

$$
\begin{aligned}
& E_t[\int_0^1\frac{1}{t}|R(\frac{\alpha}{\alpha_t}-1)|dN]^2\\
\le \;& E_t(\int_0^1\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)^2\frac{\alpha_t}{\alpha}dN)^2\\
\le \;& A_t\int_0^1\frac{1}{t^2}(\frac{\alpha_t}{\alpha}-1)^4(\frac{\alpha}{\alpha_t})^2\alpha ds \text{ by Step 1}
\end{aligned}
$$

which converges to zero by the proof of Step 2 and since $\limsup_{t\downarrow 0} A_t < \infty$.
With $-\ell_t = \int_0^1(\frac{\alpha}{\alpha_t}-1)dM_t + \int_0^1 R(\frac{\alpha}{\alpha_t}-1)dN$, Step 3 follows from

$$
\begin{aligned}
& E_t(\int_0^1(\frac{\alpha}{\alpha_t}-1)dM_t)^2 = E_t(\int_0^1(\frac{\alpha}{\alpha_t}-1)^2\alpha_t\lambda ds)\\
= \;& \int_0^1(\frac{\alpha_t}{\alpha}-1)^2\frac{\alpha}{\alpha_t}\alpha E_t\lambda ds\\
\le \;& \|\frac{\alpha}{\alpha_t}\|_\infty\|E_t\lambda\|_\infty\int_0^1(\frac{\alpha_t}{\alpha}-1)^2\alpha ds \underset{t\downarrow 0}{\longrightarrow} 0.
\end{aligned}
$$

Step 4. $\limsup_{t\downarrow 0}\int\frac{1}{t^2}\ell_t^2 dP_t \le \int(\int_0^1 v dM_\alpha)^2 dP.$
Proof. From Step 3 and Lemma 2,

$$
\|E_t\lambda - E_\alpha\lambda\|_\infty \underset{t\downarrow 0}{\longrightarrow} 0. \tag{4.12}
$$

Furthermore,

$$
\begin{aligned}
& \int_0^1[\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)]^2\alpha_t E_t\lambda ds \qquad (4.13)\\
= \;& \int_0^1[\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)]^2\alpha_t E_\alpha\lambda ds + \int_0^1[\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)]^2\alpha_t(E_t\lambda - E_\alpha\lambda)ds\\
= \;& \int_0^1[\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)]^2\{\alpha E_\alpha\lambda + (\frac{\alpha}{\alpha_t}-1)\alpha E_\alpha\lambda + \frac{\alpha}{\alpha_t}\alpha(E_t\lambda - E_\alpha\lambda)\}ds\\
=: \;& C_1 + C_2 + C_3.
\end{aligned}
$$

Here

$$|C_2| = |\int_0^1 [\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)]^2(\frac{\alpha}{\alpha_t}-1)\alpha E_\alpha \lambda ds|$$
$$\leq \|\frac{\alpha}{\alpha_t}\|_\infty \|E_\alpha \lambda\|_\infty \int_0^1 \frac{1}{t^2}|\frac{\alpha_t}{\alpha}-1|^3 \alpha ds$$
$$\leq \|\frac{\alpha}{\alpha_t}\|_\infty \|E_\alpha \lambda\|_\infty \{\int_0^1 \frac{1}{t^{2p}}(\frac{\alpha_t}{\alpha}-1)^{2p}\alpha ds\}^{1/p}\{\int_0^1 |\frac{\alpha_t}{\alpha}-1|^q \alpha ds\}^{1/q},$$

which converges to zero as in the proof of Step 2. Since

$$C_1 = \int_0^1 [\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)]^2 \alpha E_\alpha \lambda ds \underset{t\downarrow 0}{\longrightarrow} \int_0^1 v^2 \alpha E_\alpha \lambda ds \quad \text{and}$$
$$|C_3| \leq \|\frac{\alpha}{\alpha_t}\|_\infty \|E_t\lambda - E_\alpha \lambda\|_\infty \int_0^1 [\frac{1}{t}(\frac{\alpha_t}{\alpha}-1)]^2 \alpha ds \underset{t\downarrow 0}{\longrightarrow} 0$$

we obtain

$$\lim_{t\downarrow 0} \int_0^1 [\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)]^2 \alpha_t E_t \lambda ds = \int_0^1 v^2 \alpha E_\alpha \lambda ds. \tag{4.14}$$

With $-\ell_t = \int_0^1 (\frac{\alpha}{\alpha_t}-1)dM_t + \int_0^1 R(\frac{\alpha}{\alpha_t}-1)dN$ and $E_t[\int_0^1 |R(\frac{\alpha}{\alpha_t}-1)|dN]^2 \underset{t\downarrow 0}{\longrightarrow} 0$ from the proof of Step 3 we obtain Step 4 by

$$E_t[\int_0^1 \frac{1}{t}(\frac{\alpha}{\alpha_t}-1)dM_t]^2 = E_t \int_0^1 [\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)]^2 \alpha_t \lambda ds \tag{4.15}$$
$$= \int_0^1 [\frac{1}{t}(\frac{\alpha}{\alpha_t}-1)^2 \alpha_t E_t \lambda ds \underset{t\downarrow 0}{\longrightarrow} \int_0^1 v^2 \alpha E_\alpha \lambda ds.$$

From Theorem 1 we finally obtain from Steps 2 and 4 the L^2-differentiability of (P_t) in P_α with tangent vector $\int_0^1 v dM_\alpha$. □

The following corollary is immediate from Theorem 6.

Corollary 4.1 *If* $|\lambda_s(\omega)| \leq K < \infty$, $\forall s \in [0,1]$, $\omega \in \Omega$ *and* $v \in \mathcal{L}^2(\alpha\lambda\!\!\lambda^1)$ *such that for some* $(\alpha_t) \subset I$

L.1 $\limsup_{t\downarrow 0} \|\frac{\alpha_t}{\alpha} - 1\|_\infty < 1$ *and*

L.2 $\limsup_{t\downarrow 0} \frac{1}{t^\delta} \int_0^1 [\frac{1}{t}(\frac{\alpha_t(s)}{\alpha(s)}-1) - v(s)]^2 \alpha(s) ds < \infty$ *for some* $\delta > 0$,

then (P_{α_t}) *is* L^2*-differentiable in* P_α *with tangent vector* $\int_0^1 v(s)dM_\alpha(s)$.

Remark: If $\alpha_t = (1+tv)\alpha \in I$ for $0 \leq t \leq 1$, v caglad, $\|v\|_\infty < 1$, then L.1, L.2 are fulfilled and so the closed linear hull of the tangent cone $T(P_\alpha)$ established in Corollary 7 contains $V = \{\int_0^1 v(s)dM_\alpha(s); v \in L^2(\alpha\lambda\!\!\lambda^1)\}$, since the caglad functions are dense in $L^2[0,1]$. □

We next as consequence of Theorem 5 resp. Corollary 7 give more direct approach to the asymptotic efficiency of the Nelson-Aalen estimator compared to the approach in Theorem 5. The assumptions used here are technically somewhat different to the assumptions in Theorem 5 (no set of assumptions is implied by the other) but are close to each other in a practical sense.

Theorem 5 *Let $|\lambda_s(\omega)| \le K < \infty$, $\forall s$, $\forall \omega$ and assume:*

E.1 $V = \{\int_0^1 v(s)dM_\alpha(s);\ v \in L^2(\alpha\lambda\!\!\!\lambda^1)\}$ *is contained in the tangent cone established in Corollary 7;*

E.2 $\frac{1}{E_\alpha\lambda}$ *is bounded;*

E.3 $\|\frac{1}{n}\lambda_n - E_\alpha\lambda\|_\infty \underset{n\to\infty}{\longrightarrow} 0$ *in P_α-probability,*

then $\kappa : \mathcal{P} \to (D[0,1], \|\ \ \|_\infty)$, $\kappa(P_\alpha) = \dot{\int_0}\alpha(s)ds$ *is differentiable in P_α (w.r.t. the cone V) and the Nelson-Aalen estimator* $T_n = \dot{\int_0}\frac{1}{\lambda_n(s)}\, 1_{\{\lambda_n(s)>0\}}dN_{n,s}$ *is asymptotically efficient for κ in P_α (w.r.t. V).*

Proof: Step 1. κ is differentiable in P_α w.r.t. V with canonical gradient

$$\tilde{\kappa}_{\pi_t}(P_\alpha) = \int_0^t \frac{1}{E_\alpha\lambda}dM_\alpha. \tag{4.16}$$

Proof. Let $\int_0^1 vdM_\alpha \in V$ and let $\alpha_t \in I$ satisfy $\int_0^1[\frac{1}{t}(\frac{\alpha_t}{\alpha} - 1) - v]^2\alpha ds \underset{t\downarrow 0}{\longrightarrow} 0$ (which exist by E.1 and L.1) such that (P_{α_t}) has tangent vector $\int_0^1 v(s)dM_\alpha(s)$ in P_α. Then

$$\begin{aligned}
& \|\frac{1}{t}(\kappa(P_{\alpha_t}) - \kappa(P_\alpha)) - \dot{\int_0} v\alpha ds\|_\infty \\
\le\ & \int_0^1 |\frac{1}{t}(\frac{\alpha_t}{\alpha} - 1) - v|\alpha ds \\
\le\ & \{(\int_0^1 \alpha ds)\int_0^1(\frac{1}{t}(\frac{\alpha_t}{\alpha} - 1) - v)^2\alpha ds\}^{1/2} \underset{t\downarrow 0}{\longrightarrow} 0, \\
& \text{i.e.}\kappa'_{P_\alpha}(\int_0^1 vdM_\alpha) = \dot{\int_0} v\alpha ds.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
& \pi_t \circ \kappa'_{P_\alpha}(\int_0^1 vdM_\alpha) \\
=\ & \int_0^t v\alpha ds = E_\alpha(\int_0^t \frac{1}{E_\alpha\lambda}v\alpha\lambda ds) \\
=\ & E_\alpha \int_0^t \frac{1}{E_\alpha\lambda}dM_\alpha \int_0^t vdM_\alpha \\
=\ & \int \dot{\kappa}_{\pi_t}(P_\alpha)(\int_0^t vdM_\alpha)dP_\alpha,
\end{aligned} \tag{4.17}$$

where $\dot{\kappa}_{\pi_t}(P_\alpha) = \int_0^t \frac{1}{E_\alpha\lambda}dM_\alpha$ is a gradient. By assumption $\frac{1}{E_\alpha\lambda} \in L^2(\alpha\lambda\!\!\!\lambda^1)$ and so

$$\dot{\kappa}_{\pi_t}(P_\alpha) = \tilde{\kappa}_{\pi_t}(P_\alpha) = \int_0^t \frac{1}{E_\alpha\lambda}dM_\alpha \tag{4.18}$$

is the canonical gradient.

Step 2. $\|\sqrt{n}(T_n - \kappa(P_\alpha)) - n^{-1/2}\dot{\int_0}\frac{1}{E_\alpha\lambda}dM_{n,\alpha}\|_\infty \xrightarrow[n\to\infty]{P_\alpha} 0$, where $M_{n,\alpha} = \sum_{i=1}^n M_\alpha^{(i)}$.

Proof.

$$\begin{aligned}
&\|\sqrt{n}[\int_0^{\cdot}\frac{1}{\lambda_n}1_{\{\lambda_n>0\}}dN_n-\int_0^{\cdot}\alpha ds]-n^{-1/2}\int_0^{\cdot}\frac{1}{E_\alpha\lambda}dM_{n,\alpha}\|_\infty\\
\leq\;&\|n^{-1/2}\int_0^{\cdot}(\frac{n}{\lambda_n}1_{\{\lambda_n>0\}}-\frac{1}{E_\alpha\lambda})dM_{n,\alpha}\|_\infty+\sqrt{n}\int_0^1 1_{\{\lambda_n=0\}}\alpha ds.
\end{aligned}$$

The second term converges to zero, since $\|\frac{1}{n}\lambda_n-E_\alpha\lambda\|_\infty\xrightarrow{P_\alpha}0$. From the Lenglart-inequality we obtain for $\varepsilon,\eta>0$

$$\begin{aligned}
&P_\alpha(\sup_{0\leq t\leq 1}|n^{-1/2}\int_0^t(\frac{n}{\lambda_n}1_{\{\lambda_n>0\}}-\frac{1}{E_\alpha\lambda})dM_{n,\alpha}|>\varepsilon) &(4.19)\\
\leq\;&\frac{\eta}{\varepsilon^2}+P_\alpha(\int_0^1\frac{1}{n}[\frac{n}{\lambda_n}1_{\{\lambda_n>0\}}-\frac{1}{E_\alpha\lambda}]^2\alpha\lambda_n ds>\eta)\\
\leq\;&\frac{\eta}{\varepsilon^2}+P_\alpha(\|\alpha\|_\infty\|\frac{\lambda_n}{n}\|_\infty\|\frac{n}{\lambda_n}1_{\{\lambda_n>0\}}-\frac{1}{E_\alpha\lambda}\|_\infty^2>\eta)\\
\leq\;&\frac{\eta}{\varepsilon^2}+P_\alpha(\|\alpha\|_\infty K\|\frac{n}{\lambda_n}1_{\{\lambda_n>0\}}-\frac{1}{E_\alpha\lambda}\|_\infty^2>\eta).
\end{aligned}$$

Since $\|\frac{1}{E_\alpha\lambda}\|_\infty<\infty$, $E_\alpha\lambda_s\geq\delta$ for all $s\in[0,1]$ and some $\delta>0$. Therefore, for $\varepsilon>0$

$$\begin{aligned}
&P_\alpha(\|\frac{1}{\lambda_n}1_{\{\lambda_n>0\}}-\frac{1}{E_\alpha\lambda}\|_\infty\geq\varepsilon)\\
\leq\;&P_\alpha(\|\frac{n}{\lambda_n}-\frac{1}{E_\alpha\lambda}\|_\infty\geq\varepsilon,\ \|\frac{\lambda_n}{n}-E_\alpha\lambda\|_\infty<\frac{\delta}{2})\\
&+P_\alpha(\|\frac{\lambda_n}{n}-E_\alpha\lambda\|_\infty\geq\frac{\delta}{2})\\
\leq\;&P_\alpha(\|\frac{n}{\lambda_n}\|_\infty\|\frac{1}{E_\alpha\lambda}\|_\infty\|E_\alpha\lambda-\frac{\lambda_n}{n}\|_\infty\geq\varepsilon,\ \|\frac{\lambda_n}{n}-E_\alpha\lambda\|_\infty<\frac{\delta}{2})\\
&+P_\alpha(\|\frac{\lambda_n}{n}-E_\alpha\lambda\|_\infty\geq\frac{\delta}{2})\\
\leq\;&P_\alpha(\frac{2}{\delta^2}\|E_\alpha\lambda-\frac{\lambda_n}{n}\|_\infty\geq\varepsilon,\ \|\frac{\lambda_n}{n}-E_\alpha\lambda\|_\infty<\frac{\delta}{2})\\
&+P_\alpha(\|\frac{\lambda_n}{n}-E_\alpha\lambda\|_\infty\geq\frac{\delta}{2})\\
\leq\;&P_\alpha(\|E_\alpha\lambda-\frac{\lambda_n}{n}\|_\infty\geq\frac{1}{2}\delta^2\varepsilon)+P_\alpha(\|\frac{\lambda_n}{n}-E_\alpha\lambda\|\geq\frac{\delta}{2})\underset{n\to\infty}{\longrightarrow}0,
\end{aligned}$$

which implies Step 2 by (4.20). For a related derivation cf. Greenwood and Wefelmeyer (1989).

Step 3. $\sqrt{n}(T_n-\kappa(P_\alpha))\xrightarrow{\mathcal{D}}Y$, a process with continuous path's (w.r.t. P_α).

Proof. By Step 2 it is enough to check convergence of $(n^{-1/2}\int_0^{\cdot}\frac{1}{E_\alpha\lambda}dM_{n,\alpha})$ by the CLT of Rebolledo. For the predictable variation

$$\begin{aligned}
<n^{-1/2}\int_0^{\cdot}\frac{1}{E_\alpha\lambda}dM_{n,\alpha}>_t\;&=\;\int_0^t(\frac{1}{E_\alpha\lambda})^2\alpha\frac{\lambda_n}{n}ds &(4.20)\\
\underset{n\to\infty}{\xrightarrow{P_\alpha}}\int_0^t\frac{1}{E_\alpha\lambda}ds\;&=:\;\Lambda(t)\quad\text{cf. also (3.9)).}
\end{aligned}$$

The Lindeberg type condition

$$\Delta \frac{1}{\sqrt{n}} \int_0^t \frac{1}{E_\alpha \lambda} dM_{n\alpha} = \frac{1}{\sqrt{n}} \frac{1}{E_\alpha \lambda(t)} \Delta N_{n,t} \xrightarrow[n\to\infty]{P_\alpha} 0$$

is shown as in the proof of Proposition 4.

By Step 3 the sequence $(\sqrt{n}(T_n - \kappa(P_\alpha)))$ is tight (on $(D[0,1], \| \ \|_\infty)$) and (T_n) is asymptotically linear as in (3.2) which implies asymptotic efficiency. $\square$

5 LAN and Differentiability

In this section we extend the relation between differentiability and efficiency as used in Section 3 in the case of i.i.d. models to the general case of LAN models.

Let $\mathcal{P}_n = \{P_{n,\vartheta}; \vartheta \in \Theta\}$ be an asymptotic model and let $V \subset H$ be a cone in a Hilbert space H with norm $\| \ \| = \| \ \|_\vartheta$. For each $v \in V$ let $(\vartheta_{n,v}) \subset \Theta$ be a sequence in Θ with $\vartheta_{n,v} \to \vartheta$ (typically $a_n(\vartheta_{n,v} - \vartheta) \to a_v \neq 0$ for some sequence $a_n \to \infty$). Then $(\mathcal{P}_n)$ is LAN in ϑ (with rate (a_n)) if for some linear process Z_n on lin V

$$\log \frac{dP_{n,\vartheta_{nv}}}{dP_{n,\vartheta}} = Z_n(v) - \frac{1}{2}\|v\|^2 + o_{P_{n,\vartheta}}(1) \tag{5.1}$$

and $P_{n,\vartheta}^{Z_n(v)} \xrightarrow{\mathcal{D}} N(0, \|v\|^2)$, $v \in V$.

In the case of iid models $V = T(P_\vartheta) \subset L^2(P_\vartheta) = H$ and for $g \in T(P_\vartheta)$, $Z_n(g) = n^{-1/2} \sum_{i=1}^n g(X_i)$. Let $\kappa : \Theta \to B$ be a differentiable functional, i.e. $a_n(\kappa(\vartheta_{n,v} - \kappa(\vartheta)) \to \kappa'_\vartheta(v)$, $v \in V$, where $\kappa'_\vartheta : \text{lin}V \to B$ is continuous linear. While in iid models (as in Section 3) it is natural to consider the estimation of functionals κ defined directly on the basic model of underlying distributions, for the more general case of LAN models it seems to be more natural to consider the functionals κ defined on a parameter space Θ (e.g. in the case of point processes with intensities $\lambda_{n,\alpha}$ as functionals of α). Note also that the differentiability postulated here is an asymptotic form of differentiability. It is different from the differentiability of a functional in a fixed model κ as in Section 3 (in the situation of iid models). For $b^* \in B^*_\tau$ (B^*_τ denotes the class of all continuous linear functionals w.r.t. the underlying topology τ) let $\dot\kappa_{b^*}(\cdot, \vartheta) \in V$ be a gradient in direction b^*, i.e.

$$b^* \circ \kappa'_\vartheta(v) = < v, \dot\kappa_{b^*}(\cdot, \vartheta) >, \ v \in V, \tag{5.2}$$

and let $\tilde\kappa_{b^*}(\cdot, \vartheta)$ be the projection of $\dot\kappa_{b^*}(\cdot, \vartheta)$ on $\overline{\text{lin}}(V)$, the canonical gradient in direction b^*.

Assume the existence of a probability measure $N \in M^1(B, \mathcal{B})$ with separable support, such that for all $\mathcal{B}$-measurable $b^* \in B^*_\tau$, $b^*(N) = N(0, \|\tilde\kappa_{b^*}(\cdot, \vartheta)\|^2)$. An estimator sequence (T_n) is called asymptotically efficient for κ in ϑ if

$$P_{n,\vartheta}^{a_n(T_n - \kappa(P_{n,\vartheta}))} \xrightarrow{\mathcal{D}} N \tag{5.3}$$

or, equivalently,

$$(a_n(T_n - \kappa(P_{n,\vartheta}))) \tag{5.4}$$

is tight, and

$$a_n(b^* \circ T_n - b^* \circ \kappa(P_{n,\vartheta})) \xrightarrow{\mathcal{D}} N(0, \|\tilde{\kappa}_{b^*}(\cdot,\vartheta)\|^2) \tag{5.5}$$

for all $\mathcal{B}$-measurable $b^* \in B^*_\tau$. In order to obtain a sharp bound for the as. variance of estimators and the possibility of constructing efficient estimators one has to choose a sufficiently large tangent cone.

(5.4) and (5.5) are equivalent to the condition that $b^* \circ T_n$ is asymptotically efficient for the real functional $b^* \circ \kappa$, which in turn is wellknown to be equivalent to a stochastic expansion

$$a_n(b^* \circ T_n - b^* \circ \kappa(P_{n,\vartheta})) = Z_n(\tilde{\kappa}_{b^*}(\cdot,\vartheta)) + o_{P_{n,\vartheta}}(1). \tag{5.6}$$

In the case that $B = (D[0,1], \| \ \|_\infty)$, (5.4), (5.5) are equivalent to:

$$\pi_t \circ T_n \text{ is asymptotically efficient for } \pi_t \circ \kappa,\ t \in [0,1]. \tag{5.7}$$

An estimator sequence (T_n) is called <u>regular</u> if for some $L \in M^1(B, \mathcal{B})$

$$P_{n,\vartheta_{n,v}}^{b^*(a_n(T_n - \kappa(P_{n,\vartheta_{n,v}})))} \to L^{b^*},\ b^* \in B^*_\tau,\ b^*\ \mathcal{B} - \text{measurable}. \tag{5.8}$$

Again in the case of $B = D[0,1]$ one can restrict to $b^* = \pi_t,\ t \in [0,1]$. The following version of the convolution theorem can be proved analogously to the proof of Theorems 3.14, 3.7 in van der Vaart (1988). This theorem justifies the notion of asymptotic efficiency in (5.3). Let for $B' \subset B^*$, $\mathcal{R}(B') := \bigcup_{A \subset B',\, A \text{ finite}} \sigma_A$ denote the cylinder σ-algebra on B (σ_A the σ-algebra generated by A).

Theorem 6 *If V is convex, $\kappa : \Theta \to B$ is differentiable in ϑ with canonical gradient $\tilde{\kappa}_{b^*}(\cdot,\vartheta)$ in direction $b^* \in B^*$, then:*

a) For any limit point L of a regular estimator sequence (T_n), $L = N * M$ is the convolution of two cylinder measures on $(B, \mathcal{R}(B^*))$ with $b^*(N) = N(0, \|\tilde{\kappa}_{b^*}(\cdot,\vartheta)\|^2)$, $b^* \in B^*$.

b) If L is $\tau(B^*)$-tight, then there exist extensions of N, M to probability measures on (B, σ_{B^*}).

B^* can be replaced by a separating subspace $B' \subset B^*$. The tightness condition in b) is fulfilled generally, if (B, τ) is a polish top. vectorspace and $B' = B^*$, or if $B = (D[0,1], \| \ \|_\infty)$ and $B' =< \{\pi_t;\ t \in [0,1]\} >$ or if $B = A^*$, $(A, \| \ \|)$ a normed space with $B' = A$; so e.g. in the reflexive banach spaces, ℓ_∞, L_∞. Similarly, also the version of the minimax theorem (cf. Theorem 3.17 in [21]) extends to the general case.

Let for $B' \subset B^*_\tau$ and $N \in M^1(B, \mathcal{B})$, $\mathcal{L}(B', N)$ denote the class of all measurable loss functions $\ell : B \to \mathbb{R}^1$ such that for some sequence (ℓ_k) of cylinder functions $\ell_k \leq \ell$ and $\ell_k \uparrow \ell[N]$. In particular if B is a metrisable, locally convex topological vectorspace and $\ell : B \to \mathbb{R}^1$ is subconvex (i.e. $\ell(0) = 0 \leq \ell(b)$, $\ell(b) = \ell(-b)$ and $\{b \in B : \ell(b) \leq c\}$ is convex and τ-closed for $c \in \mathbb{R}_+$), then $\ell \in \mathcal{L}(B^*, N)$.

Theorem 7 *If V is convex, $\kappa : \Theta \to B$ is differentiable in ϑ and if there exists $N \in M^1(B, \sigma_{B'})$ with $b'(N) = N(0, \|\tilde{\kappa}_{b'}(\cdot, \vartheta)\|^2), \forall b' \in B'$, a point separating subspace of B^*, then for all $\ell \in \mathcal{L}(B', N)$ and for all estimator sequences (T_n) holds:*

$$\lim_{c\to\infty} \underline{\lim} \sup_{\vartheta_n \in B_n(c)} E_{\vartheta_n} \ell(\sqrt{n}(T_n - g(\vartheta_n))) \geq \int \ell(b) dN(b), \tag{5.9}$$

where

$$B_n(c) = \{\vartheta_n = \vartheta_{n,v}; \|v\| \leq c\}.$$

Define $\phi : B_1 \to B_2$ to be a <u>Hadamard differentiable function, tangentially to S</u>, if for $b \in B_1$, $h_n \in B_1$, $h_n \to h \in S$, $t_n \to 0$, there exists a continuous linear function $\phi_b' : B_1 \to B_2$ such that

$$\frac{1}{t_n}(\phi(b + t_n h_n) - \phi(b)) \to \phi_b'(h). \tag{5.10}$$

The following result extends Theorem 4.11 of van der Vaart (1988). Let S denote the separable support of N.

Theorem 8 *If $\kappa : \Theta \to B_1$ is differentiable in ϑ and if $\phi : B_1 \to B_2$ is Hadamard-differentiable in $\kappa(\vartheta)$ tangentially to lin$\{S, \kappa_\vartheta'(V)\}$ with $\mathcal{B}_1 - \mathcal{B}_2$-measurable derivative, then $\phi \circ \kappa$ is differentiable in ϑ. If (T_n) is asymptotically efficient for κ in ϑ and $P_{n,\vartheta}^{\sqrt{n}(T_n - \kappa(\vartheta))} \to N$ and $\phi \circ T_n$ is $\mathcal{B}_2$-measurable, then $(\phi \circ T_n)$ is asymptotically efficient for $\phi \circ \kappa$ in ϑ.*

In the next step we apply these results to models with multiplicative intensities (which are however not necessarily iid models). The following LAN theorem is due to Dzapharidze (1985), for a simplified proof cf. also Greenwood and Wefelmeyer (1989).

Theorem 9 *Let for $\alpha \in I$ there exist a bounded function $\Lambda : [0,1] \to \mathbb{R}^1$ and $a_n \to \infty$, such that $\frac{1}{a_n^2}\lambda_n \to \Lambda$ uniformly in $P_{n,\alpha}$ probability. Let $v \in L^2(\alpha\lambda\!\!\lambda^1)$ and $\alpha_{n,v} \in I$ satisfy*

$$\int_0^1 [a_n((\frac{\alpha_{n,v}(s)}{\alpha(s)})^{1/2} - 1) - \frac{1}{2}v(s)]^2 \alpha(s) ds \to 0, \tag{5.11}$$

then

$$\log(\frac{dP_{n,\alpha_{n,v}}}{dP_{n\alpha}}) = \frac{1}{a_n}\int_0^1 v(s) dM_{n\alpha}(s) - \frac{1}{2}\int_0^1 v^2(s)\alpha(s)\Lambda(s) ds + o_{P_{n\alpha}}(1) \tag{5.12}$$

and

$$P_{n,\alpha}^{\frac{1}{a_n}\int_0^1 v(s) dM_{n,\alpha}(s)} \xrightarrow{\mathcal{D}} N(0, \int_0^1 v(s)^2 \alpha(s)\Lambda(s) ds, \tag{5.13}$$

i.e. we have LAN with central sequence

$$Z_n(v) = \frac{1}{a_n}\int_0^1 v(s) dM_{n\alpha}(s). \tag{5.14}$$

Let $V(\alpha) := \{v \in L^2(\alpha);\ v$ is a tangent vector of a sequence in $I\} \subset L^2(\alpha\Lambda)$.

The following result extends Example 4.8 of Greenwood and Wefelmeyer (1988) who considered the estimation of real functionals.

Let $\hat{A}_n = \int_0^{\cdot} \frac{1}{\lambda_n(s)} 1_{\{\lambda_n(s)>0\}} dN_n(s)$ be the Nelson-Aalen estimator for the integrated intensity κ, $\kappa(\alpha)_t = \int_0^t \alpha(s)ds$, and let $\phi : D[0,1] \to B_2$ be Hadamard-differentiable with $\mathcal{B} - \mathcal{B}_2$-measurable derivative and assume that $\phi \circ \hat{A}_n$ is measurable. We give here a direct proof of the efficiency of $\hat{A}_n$ for the estimation of κ. The method of this proof is the same as in the proof of Theorem 8. We, therefore, only indicate the necessary changes due to the different assumptions and frame work. Our functional version of this efficiency result then by Theorem 11 immediately implies the efficiency of $\phi \circ \hat{A}_n$ for the functional $\phi \circ \kappa$.

Theorem 10 *Additional to the assumptions in Theorem 12 assume that $\overline{lin\, V(\alpha)} = L^2(\alpha)$ and that $\frac{1}{\Lambda}$ is bounded. Then $(\phi(\hat{A}_n))$ is asymptotically efficient for $\phi \circ \kappa$, where $\kappa(\alpha) = \int_0^{\cdot} \alpha(s)ds$ is the integrated intensity.*

Proof: Step 1. $\kappa(\alpha) = \int_0^{\cdot} \alpha(s)ds$ is differentiable in α with canonical gradient $\tilde{\kappa}_{\pi_t}(\cdot, \alpha) = 1_{[0,1]}\frac{1}{\Lambda}$.

Proof. For $t \in [0,1]$, $v \in V(\alpha)$ holds:

$$\begin{aligned}
& |a_n(\kappa_t(\alpha_{n,v}) - \kappa_t(\alpha)) - < v, 1_{[0,t]}\frac{1}{\Lambda} >_{\alpha\Lambda} | \\
= & \ |\int_0^t (a_n(\alpha_{nv} - \alpha) - v\alpha)ds| \le \int_0^t |a_n(\alpha_{nv} - \alpha) - vd|ds \\
= & \int_0^t |a_n[(\frac{\alpha_{nv}}{\alpha})^{1/2} - 1]^2\alpha + 2a_n[(\frac{\alpha_{nv}}{\alpha})^{1/2} - 1]\alpha - v\alpha|ds \\
\le & \ \frac{1}{a_n}\int_0^t a_n^2[(\frac{\alpha_{nv}}{\alpha})^{1/2} - 1]^2\alpha ds + 2\int_0^t |a_n[(\frac{\alpha_{nv}}{\alpha})^{1/2} - 1] - \frac{1}{2}v|\alpha ds \\
= & \ o(1) \quad \text{by (5.10) uniformly in } t.
\end{aligned}$$

Step 2. $a_n[\int_0^t \frac{1}{\lambda_n}1_{\{\lambda_n>0\}}dN_n - \int_0^t \alpha ds] = \frac{1}{a_n}\int_0^t \frac{1}{\Lambda}dM_{n\alpha} + o_{P_{n,\alpha}}(1)$.

Proof.

$$\begin{aligned}
& a_n[\int_0^t \frac{1}{\lambda_n}1_{\{\lambda_n>0\}}dN_n - \int_0^t \alpha ds] \\
= & \ a_n \int_0^t \frac{1}{\lambda_n}1_{\{\lambda_n>0\}}dM_{n\alpha} - a_n\int_0^t \alpha 1_{\{\lambda_n=0\}}ds \\
= & \ \frac{1}{a_n}\int_0^1 1_{[0,t]}\frac{1}{\Lambda}dM_{n\alpha} + \frac{1}{a_n}\int_0^t (\frac{a_n^2}{\lambda_n}1_{\{\lambda_n>0\}}\frac{1}{\Lambda})dM_{n\alpha} - a_n\int_0^t \alpha 1_{\{\lambda_n=0\}}ds \\
= & \ \frac{1}{a_n}\int_0^1 1_{[0,t]}\frac{1}{\Lambda}dM_{n\alpha} + o_{P_{n\alpha}}(1) \quad \text{uniformly in } t.
\end{aligned}$$

The last equality can be proved as in the proof of Step 2 of Theorem 8 using boundedness of $\frac{1}{\Lambda}$.

Step 3. $a_n(\hat{A}_n - \kappa(P_\alpha)) \xrightarrow{\mathcal{D}} Y$ a continuous Gaussian martingale. This follows from Step 2 and the CLT of Rebolledo.

Together from Steps 1 - 3, $\hat{A}_n$ is efficient for $\kappa(\alpha)$. The asymptotic efficiency of $\phi \circ \hat{A}_n$ is a consequence of Theorem 11. $\square$

We next consider general point process models which are not necessarily with multiplicative intensities. Let for $\alpha \in I, \lambda_{n\alpha}$ be the intensity of N_n w.r.t. $P_{n\alpha}$. The following LAN result is due to Dzapharidze (1985). We give a simplified proof of this result which is a modification of the proof of Greenwood and Wefelmeyer (1989) for the case of multiplicative intensities.

Theorem 11 *Let for $\alpha \in I$ there exist a bounded function $\Lambda_\alpha : [0,1] \to \mathbb{R}^1$ and $(a_n) \subset \mathbb{R}^1$, $a_n \to \infty$ such that*

1. *$\frac{1}{a_n^2}\lambda_{n\alpha} \to \Lambda_\alpha$ uniformly in $P_{n\alpha}$-probability;*
2. *for some $v \in L^2(\Lambda_\alpha ds)$ and $(\alpha_{n,v}) \subset I$ holds:*
$$\int_0^1 [a_n((\frac{\lambda_{n,\alpha_{n,v}}(s)}{\lambda_{n,\alpha}(s)})^{1/2} - 1) - \frac{1}{2}v(s)]^2 \Lambda_\alpha(s) ds = o_{P_{n,\alpha}}(1),$$
then
$$\log \frac{dP_{n,\alpha_{n,v}}}{dP_{n,\alpha}} = \frac{1}{a_n}\int_0^1 v(s) dM_{n\alpha}(s) - \frac{1}{2}\int_0^1 v^2(s)\Lambda_\alpha(s) ds + o_{P_{n,\alpha}}(1) \qquad (5.15)$$
and
$$\frac{1}{a_n}\int_0^1 v(s) dM_{n,\alpha}(s) \xrightarrow{\mathcal{D}} N(0, \int v^2(s)\Lambda_\alpha(s) ds). \qquad (5.16)$$

Proof: Let $R(x) = \log(1+x) - x + \frac{x^2}{2}$, then by some calculations with $\alpha_n = \alpha_{n,v}$
$$\begin{aligned}
\log \frac{dP_{n,\alpha_{n,v}}}{dP_{n,\alpha}} &= 2\int_0^1 [(\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1] dM_{n,\alpha} \\
&- 2\int_0^1 [(\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1]^2 \lambda_{n,\alpha} ds \\
&- \int_0^1 [(\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1]^2 dM_{n,\alpha} + 2\int_0^1 R((\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1) dN_n.
\end{aligned}$$

<u>Step 1.</u> For any $\varepsilon > 0$:
$$\int_0^1 1_{\{|(\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1| > \varepsilon\}} \lambda_{n,\alpha} ds = o_{P_{n,\alpha}}(1), \qquad (5.17)$$
$$\int_0^1 [((\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1]^2\, 1_{\{|(\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1| > \varepsilon\}} \lambda_{n,\alpha} ds = o_{P_{n,\alpha}}(1). \qquad (5.18)$$

<u>Proof.</u> Let $A_n := ((\frac{\lambda_{n,\alpha_n}}{\lambda_{n,\alpha}})^{1/2} - 1]$, then
$$\begin{aligned}
&\int_0^1 1_{\{|A_n| > \varepsilon\}} \lambda_{n,\alpha} ds \le \frac{1}{\varepsilon^2}\int_0^1 A_n^2\, 1_{\{|A_n| > \varepsilon\}} \lambda_{n,\alpha} ds \\
&= \frac{1}{\varepsilon^2}\int_0^1 (a_n A_n)^2\, 1_{\{|A_n| > \varepsilon\}} \Lambda_\alpha ds + o_{P_{n,\alpha}}(1) \\
&= \frac{1}{\varepsilon^2}\int_0^1 \frac{1}{4} v^2\, 1_{\{|A_n| > \varepsilon\}} \Lambda_\alpha ds + o_{P_{n,\alpha}}(1) \\
&= o_{P_{n,\alpha}}(1).
\end{aligned}$$

Step 2.

$$2\int_0^1 A_n dM_{n,\alpha} - \frac{1}{a_n}\int_0^1 v dM_{n,\alpha} = o_{P_{n,\alpha}}(1). \tag{5.19}$$

Proof. Consider the predictable variation

$$\begin{aligned} < \int_0^{\cdot}(A_n - \frac{v}{a_n})dM_{n,\alpha} >_1 &= \frac{1}{a_n^2}\int_0^1 (a_n A_n - \frac{1}{2}v)^2 \lambda_{n,\alpha} ds \\ &= \int_0^1 (a_n A_n - \frac{1}{2}v)^2 \Lambda_\alpha ds + o_{P_{n,\alpha}}(1) = o_{P_{n,\alpha}}(1). \end{aligned}$$

Therefore, (5.18) follows from the Lenglart-inequality.

Step 3.

$$2\int_0^1 A_n^2 \lambda_{n\alpha} ds - \frac{1}{2}\int_0^1 v^2 \Lambda_\alpha ds = o_{P_{n,\alpha}}(1). \tag{5.20}$$

Proof.

$$\begin{aligned} &|\int_0^1 A_n^2 \lambda_{n,\alpha} ds - \frac{1}{4}\int_0^1 v^2 \Lambda_\alpha ds| \\ \le & \int_0^1 (a_n A_n)^2 |\frac{\lambda_{n\alpha}}{a_n^2} - \Lambda_\alpha| ds + \int_0^1 ((a_n A_n)^2 - \frac{1}{4}v^2)\Lambda_\alpha ds \\ = & \ o_{P_{n,\alpha}}(1). \end{aligned}$$

Step 4.

$$\int_0^1 R(A_n) dN_n = o_{P_{n,\alpha}}(1). \tag{5.21}$$

Proof. By (5.16) with Lenglarts inequality

$$\int_0^1 1_{\{|A_n|>\varepsilon\}} dM_{n,\alpha} = o_{P_{n,\alpha}}(1).$$

With $dN_n = dM_{n,\alpha} + \lambda_{n,\alpha} ds$, therefore, again using (5.16)

$$P_{n,\alpha}(\int_0^1 1_{\{|A_n|>\varepsilon\}} dN_n \ge 1) \to 0. \tag{5.22}$$

This implies

$$\int_0^1 R(A_n)\, 1_{\{|A_n|>\varepsilon\}} dN_n = o_{P_{n,\alpha}}(1)$$

and, therefore, also for some sequence $\varepsilon_n \to 0$

$$\int_0^1 R(A_n)\, 1_{\{|A_n|>\varepsilon_n\}} dN_n = o_{P_{n,\alpha}}(1).$$

Since for $|x| \le \frac{1}{2}$, $R(x) \le 2|x|^3$,

$$\begin{aligned} &\int_0^1 R(A_n)\, 1_{\{|A_n|\le\varepsilon_n\}} dN_n \\ \le & \ 2\varepsilon_n^3 \int_0^1 1_{\{|A_n|\le\varepsilon_n\}} dM_{n,\alpha} \\ + & \ 2\varepsilon_n^3 \int_0^1 1_{\{|A_n|\le\varepsilon_n\}} \lambda_{n,\alpha} ds = o_{P_{n,\alpha}}(1) \end{aligned}$$

by (5.16) and Lenglarts inequality.

Step 5.

$$\int_0^1 A_n^2 dM_{n,\alpha} = o_{P_{n,\alpha}}(1). \tag{5.23}$$

Proof. From (5.21): $\int_0^1 A_n^2 1_{\{|A_n|>\varepsilon\}} dN_n = o_{P_{n,\alpha}}(1)$ and so by (5.18) $\int_0^1 A_n^2 1_{\{|A_n|>\varepsilon\}} dM_{n,\alpha} = o_{P_{n,\alpha}}(1)$. On the other hand

$$\begin{aligned} & < \int_0^{\cdot} A_n^2 1_{\{|A_n|\leq\varepsilon_n\}} dM_{n,\alpha} >_1 \\ = & \int_0^1 A_n^4 1_{\{|A_n|\leq\varepsilon_n\}} \lambda_{n,\alpha} ds \\ \leq & \ \varepsilon_n^2 \int_0^1 A_n^2 1_{\{|A_n|\leq\varepsilon_n\}} \lambda_{n,\alpha} ds \\ = & \ o_{P_{n,\alpha}}(1) \quad \text{by (5.18).} \end{aligned}$$

Step 6.

$$\frac{1}{a_n}\int_0^1 v dM_{n,\alpha} \to N(0, \int_0^1 v^2 \Lambda_\alpha ds). \tag{5.24}$$

Proof. This follows from the CLT of Rebolledo, since the predictable variation converges

$$< \frac{1}{a_n}\int_0^{\cdot} v dM_{n,\alpha} >_t = \int_0^t v^2 \frac{\lambda_{n,\alpha}}{a_n^2} ds \to \int_0^t v^2 \Lambda_\alpha ds.$$

Also the Lindeberg condition is satisfied

$$\left|\int_0^1 \frac{v}{a_n} 1_{\{|\frac{v}{a_n}|>\varepsilon\}} \lambda_{n,\alpha} ds\right| = o_{P_{n,\alpha}}(1).$$

□

Again as consequence one obtains asymptotic optimality of martingale estimators

$$T_n = \frac{1}{a_n}\int_0^{\cdot} h \, dN_n \quad \text{for} \quad E_\alpha \int_0^{\cdot} h \, \Lambda_\alpha ds = \kappa(\alpha)$$

if the tangent cone $V(\alpha)$ is big enough (as in Theorem 13). By Theorem 11 this also implies the asymptotic efficiency of differentiable functionals $\phi(T_n)$ as estimators of $\phi \circ \kappa$. An interesting application is to kernel type estimators of a smoothed intensity as considered by Ramlau-Hansen (1983). The intensity α itself is not a differentiable functional of the integrated intensity $\kappa(\alpha) = \int_0^{\cdot} \alpha(s) ds$ and so we cannot obtain an efficient estimator for α as consequence of Theorems 11 and 14.

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References

[1] Aalen, O. O.: Nonparametric inference for a family of counting processes. Ann. Statist. 6 (1978), 701 - 726

[2] Bremaud, P.: Point Processes and Queues: Martingale Dynamics. Springer, 1981

[3] Csiszar, I.: A note on Jensen's inequality. Studia Scient. Math. Hung. 1 (1966), 185 - 188

[4] Dzhaparidze, K.: On asymptotic inference about intensity parameters of a counting process. Bull. Int. Statist. Inst. 51, (1985), Vol. 4, 23.2-1 - 23.2-15

[5] Gill, R. D.: Non- and semi-parametric maximum likelihood estimators and the von Mises method. Scand. J. Statistics 16 (1989), 97 - 128

[6] Greenwood, P. E. and Wefelmeyer, W.: Efficiency bounds for estimating functionals of stochastic processes. Preprint, 1989

[7] Hahn, M. G.: Central limit theorems in $D[0,1]$. Zeitschrift W.-theorie verw. Gebiete 44 (1978), 89 - 101

[8] Holtrode, R.: Asymptotische Effizienz des Nelson-Aalen Schätzers. Diplomarbeit, Münster, 1990

[9] Jacod, J.: Multivariate point processes, predictable projections, Radon-Nikodym derivatives representation martingales. Zeitschrift W.-theorie verw. Gebiete 31 (1975), 235 - 253

[10] Jacod, J.: Regularity, partial regularity, partial information process for a filtered statistical model. Prob. Theory and Related Fields 86 (1990), 305 - 335

[11] Jacod, J. and Shiryaev, A. N.: Limit Theorems for Stochastic Processes. Springer, 1987

[12] Karr, A. F.: Point Processes and Their Statistical Inference. M. Dekker, 1985

[13] Kutoyants, Y. A.: Parameter Estimation for Stochastic Processes. Heldermann, 1984

[14] Kutoyants, Y. A. and Liese, F.: Minimax bounds for estimations of intensity of Poisson processes. Preprint, 1990

[15] Le Cam, L.: Asymptotic Methods in Statistical Decision Theory. Springer, 1986

[16] Liese, F.: Estimation of intensity measures of Poisson point processes. Preprint, 1990

[17] Liptser, R. S. and Shiryayev, A. N.: Statistics of Random Processes II. Applications. Springer, 1978

[18] Millar, P. W.: Optimal estimation in the non-parametric multiplicative intensity model. Preprint, 1988

[19] Pfanzagl, J. and Wefelmeyer, W.: Contributions to a General Asymptotic Statistical Theory. Lecture Notes in Statistics 13. Springer, 1982

[20] Pollard, D.: Convergence of Stochastic Processes. Springer Series in Statistics, 1984

[21] Ramlau-Hansen, H.: Smoothing counting process intensities by means of kernel functions. Ann. Statist. 11 (1983), 453 - 466

[22] Rebolledo, R.: Sur les applications de la théorié des martingales à l'ètude statistique d'une famille de processus ponctuels. Lect. Notes Math. 636 (1978), 27 - 70

[23] Rüschendorf, L.: Inference for random sampling processes. Stoch. Processes Appl. 32 (1989), 129 - 140

[24] Strasser, H.: Mathematical Theory of Statistics. de Gruyter, 1985

[25] van der Vaart, A. W.: Statistical estimation in large parameter spaces. CWI Tract 44, 1988

[26] Wellner, J. A.: Asymptotic optimality of the product limit estimator. Ann. Statist. 10 (1982), 595 - 602

[27] Witting, H.: Mathematische Statistik I. Teubner, 1985

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Differentiability of Point Process Models and Asymptotic Efficiency of Differentiable Functionals

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